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Dynamic modeling of robotic manipulators for accuracy evaluation

Stefanie A. Zimmermann[1], Tobias F. C. Berninger[2], Jeroen Derkx[1], Daniel J. Rixen[2]

Abstract— **In order to fulfill conflicting requirements in the development of industrial robots, such as increased accuracy of a weightreduced manipulator with lower mechanical stiffness, the robot's dynamical behavior must be evaluated early in the development process. This leads to the need of accurate multibody models of the manipulator under development.**

This paper deals with multibody models that include flexible bodies, which are exported from the corresponding Finite Element model of the structural parts. It is shown that such a flexible link manipulator model, which is purely based on development and datasheet data, is suitable for an accurate description of an industrial robot's dynamic behavior. No stiffness parameters need to be identified by experimental methods, making this approach especially relevant during the development of new manipulators. This paper presents results of experiments in time and frequency domain for analyzing the modeling approach and for validating the model performance against real robot behavior.

I. INTRODUCTION

Development of new types of manipulators requires an early investigation of the robot's dynamical behavior to, for example, optimize the design of the structure, or to develop model-based control. In order to calculate and evaluate the dynamic performance for different application scenarios, accurate multi-body models of the manipulator under development are needed.

Creating a dynamic manipulator model is usually accompanied by the estimation or identification of parameters for the multibody components (e. g. springs and dampers). A common way to find these parameters is by fitting the model to measurements gained from the corresponding real robot. The drawback of such experimental based methods is that the contribution from each individual element of the robot is hard to deduce, since elasticity from multiple elements are often lumped together. To be useful during development, the manipulator model should furthermore be based on information that is available to the designer, without having a prototype available. This includes catalogue data on the used components, specific test measurements for isolated components and the geometrical data of the manipulator structure.

Most of the dynamic manipulator models proposed in literature follow the classical multibody approach that assumes the links to be rigid and the joints to behave like linear spring-damper pairs with ideal transmission characteristics (see e. g. [23], [19], [1]). This simple model is extended in this paper by including flexible bodies. The motivation is the possibility to obtain the link components of the multibody model from the Finite Element data, which is available during development. A parameter estimation to fit the manipulator's flexible behavior is therefore not needed. Furthermore, the structural dynamics of the links is included in the multibody model, resulting in a high-fidelity manipulator model.

[1]Stefanie A. Zimmermann and Jeroen Derkx are with ABB AB Robotics, Zurich, Switzerland stefanie.zimmermann@de.abb.com
[2]Tobias F. C. Berninger and Daniel J. Rixen are with the Chair of Applied Mechanics, Technical University of Munich, Germany t.berninger@tum.de

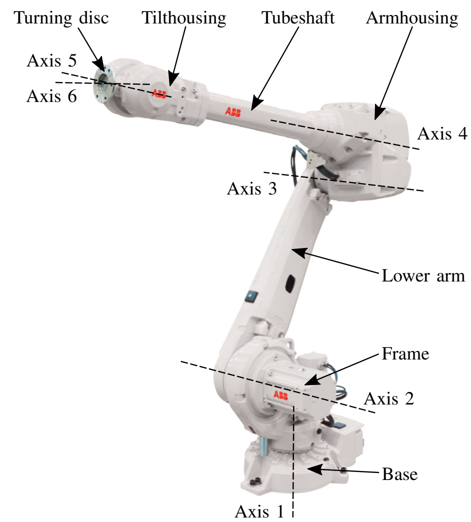


Fig. 1: 6-axes industrial robot.

The dynamic behavior of a manipulator is also highly influenced by the joints, which connect its links (see e. g. [1]). Joint flexibilities are common in current industrial robots, when harmonic drives or compact gears are used. Such components have gained wide acceptance because of their compact design, light weight and high reduction ratios, but they also introduce considerable flexibilities in the drivetrain [16]. More detailed models of robotic joints therefore consider flexibilities caused by the gear wheels and shafts, as well as by the bearings.

II. DYNAMIC MODELING OF ROBOT MANIPULATORS

Dynamic models of a manipulator describe the relation between the actuation and acting contact forces, and the resulting acceleration and motion trajectories [20]. A mechanical system can often be seen as the sum of a number of more or less stiff elements [11]. Such a system is called Multibody System (MBS) and is characterized by two distinguishing features [4]: the overall motions of the MBS's bodies are finite and "large", and all bodies are connected by mechanical joints that impose restrictions on their relative motion. Depending on the intended use of the dynamic model, the bodies can be assumed as rigid, elastically coupled, or elastically deformable.

A. Rigid Multibody Systems

Rigid MBSs consist of rigid bodies and stiff, massless joints only. The mass and inertia of the actuators and gearboxes are added to the corresponding link parameters. Rigid body models posses in principle as many degrees of freedom (DOFs) as they are defined by the joints. The rigid body model of an industrial robot with six serial rotational axes (see Fig. 1) has, for example, six DOFs in total. A schematic drawing of this model is shown in Fig. 2a.

Taking into account the increasing operational speed of contemporary robots, and the demand for high payload capacity with lightweight arm structures, a more detailed and realistic system model is needed. Two different approaches are common: The manipulator's flexibilities can be concentrated either at the transmissions' level, or at the links' level.

B. Lumped parameter Multibody Systems

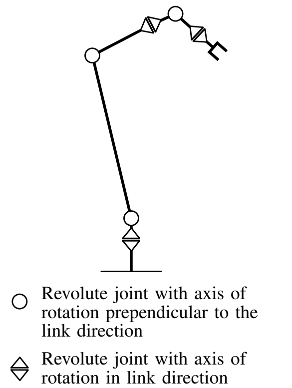
Lumped parameter MBSs add elastic coupling, i. e. add DOFs, to a rigid body model using so called flexible joints [4]. The model parameters, such as spring or damping constants, can be tuned to better fit the reality.

Such models are based on the assumption that the main elasticity occurs in the drive chain between the motors and the links. In simple dynamic modeling approaches, flexible joints are often modeled as linear torsional springs (see e. g. [23], [13]). Another common approach to model joint elasticities are torsional spring-damper pairs [19], or three-dimensional, orthogonally oriented, spring-damper pairs (see example in Fig. 2b). While the rigid body parameters are known from the corresponding CAD model, the lumped stiffness parameters need to be identified. The identification can be carried out in the frequency domain by searching for the minimal sum of weighted differences between the experimental frequency response functions (FRFs) and the FRFs obtained from the model.

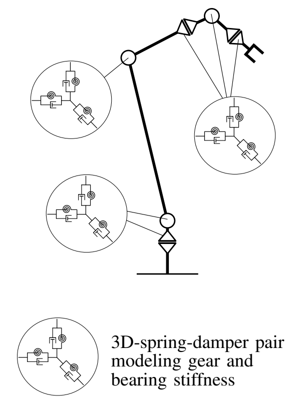
C. Elastic Multibody Systems

Elastic MBSs include elastic bodies in addition to the lumped modeled joint elasticities in order to take into account high-frequency structural modes [11]. Robotic systems with flexible links are "continuous dynamical systems characterized by an infinite number of DOFs and are governed by nonlinear, coupled, ordinary, and partial differential equations" [22]. Due to the fact, that the exact solution of such a mathematical model is not feasibly practical, methods that capture the most essential flexibilities, using a finite number of parameters, are required. Three different approaches to derive approximated finite dimensional models are generally used [3], [22]:

Finite element models are the most accurate models, describing the link deflection by the use of a large number of DOFs. The solution domain is first divided into a finite number of sub-domains called Finite Elements (FEs). The solution within each FE is then approximated by a small number of continuous functions that are based on values at discrete points, the so called nodes. To guarantee the continuity of the solution across FEs, neighboring elements share common nodes, i. e. share common DOFs. Due to their complexity, FE models are normally only used in the mechanical design of robotic systems and not as a tool to develop models suitable for dynamic simulation and control.



(a) Rigid body model of a 6-axes industrial robot with six DOFs.



(b) Example of a lumped parameter model (24 DOFs) of a 6-axes industrial robot.

Fig. 2: Rigid multibody model and example of a lumped parameter model of a 6-axes industrial robot.

Assumed modes models are derived from the partial differential equation formulation by modal truncation. This description requires to find out the best selection for link boundary conditions, which makes this conceptually simple approach challenging. The assumed modes approach is used in the context of dynamic substructuring and component mode synthesis (see Sec. III).

Lumped parameter models divide each flexible link into a finite number of rigid links, introducing pseudojoints. Elasticity is modeled as springs that restrict the motion of each pseudojoint. The lumped parameter approach is an easy way to develop a model that is simple enough to be used for real-time control and for dynamic evaluation of a manipulator. However, the method is rarely used because of the difficulty in determining the spring constants of the pseudojoints.

A manipulator model with three links among them two are modeled as flexible bodies is presented in [25]. These flexible links are assumed to be slender and homogeneous, and are treated as simple beams, which can move in three dimensions. A dynamic modeling approach that takes into account both flexible links and flexible joints is given in [21]. A flexible joint is modeled as a linear torsional spring, the link flexibility is described by adding the link deformation to the rigid body motion. The assumed modes discretisation technique is applied to derive the dynamic equations of motion.

The flexible link model presented in this paper consists of seven flexible bodies connected by six flexible joints with six

DOFs each (see Sec. V). The flexible bodies are described by reduced representations of their FE models and connected using individually defined interface nodes (see Sec. IV).

III. Reduction of FE-Models

The number of DOFs of a flexible body, which is described by the FE formulation, needs to be reduced in order to efficiently include it in a multibody model.

One of the most common approaches to model flexible MBSs is based on the concept of floating frames [17]. This approach divides the total motion of the flexible body into two parts: rigid body motions, which are represented by the motion of the floating frame, and superimposed elastic motions described with respect to this frame [4]. The approach allows the assumption that the elastic motions cause infinitesimal deformations, although the total motion is always finite. A robotic manipulator, for instance, undergoes large rigid body motions but the elastic deformations of its links remain small. The Dynamic Substructuring technique is based on the floating frame of reference approach, and allows to derive the behavior of the entire structure from its constituent components [7].

For solving very large structural dynamics problems, some form of component mode synthesis (CMS) is commonly employed for dynamic analysis. CMS techniques are based on the idea of substructuring using modes to represent the dynamic behavior of a substructure [6]. Usually, the CMS method comprises some kind of modal analysis on the substructure. The obtained modal vectors are used to reduce the full set of physical coordinates to a smaller set of generalized coordinates expressed in the modal domain [12].

A technique that has frequently been employed to reduce a system's number of DOFs is referred to as the Guyan reduction method [9], [10]. A subset of the coordinates is selected arbitrarily as the set of master coordinates, and the remaining coordinates are slave coordinates [7]. This approach assumes the response between eliminated and retained DOFs correctly represented by the static modes only, since the dynamic contribution of the slave DOFs is neglected. The accuracy of the frequencies obtained by the Guyan reduction method depends on the selection of master DOFs, i. e. the choice of retained coordinates. In the models presented in Sec. V, only the DOFs, which are needed to connect the substructures to the surrounding (referred to as interface nodes), serve as master nodes. The dynamic representation may be improved by retaining additional physical DOFs that are not required to connect the substructure to other model components. Various methods for selecting such additional DOFs have been proposed (see e. g. [18]).

Bampton and Craig have presented a method for reducing the total number of DOFs of a structure while retaining accurate description of its dynamic behavior [2]. The so called Craig-Bampton (CB) method uses not only static modes but also vibration modes associated with the dynamics of the DOFs internal to the substructures (slave DOFs) when the master DOFs are fixed. The accuracy depends not only on the selection of master DOFs but also on the selection of retained modes, since not all eigenmodes have the same influence on the solution of a specific problem. To obtain an accurate reduced model for general load distributions, the retained modes should agree with the modes excited during operation, e. g. up to some given limit [8].

IV. Including flexiblebodies in a multibody environment

A. *Standard input data*

In order to include flexible bodies in a multibody simulation environment (e. g. Dymola), the so-called Standard Input Data (SID) can be used, which contains the relevant information of the bodies' dynamic behavior. Using the floating frame of reference approach, a set of data for each flexible body's kinematic and dynamic equations is gained, which is based on the modal representation of small body deformations. This allows a pre-calculation of the integrals, which appear in the multibody system equations, and the results can be stored in the SID-file before simulation.

B. *Process of model order reduction*

Applying the idea of dynamic substructuring, the manipulator is divided into its natural components, i. e. the six links and the base. In order to include a substructure as a flexible body in the multibody environment of Dymola, the number of DOFs of the original FE model needs to be reduced, and a SID-file must be created. The general process to include the FE model of a substructure in a multibody simulation can be divided in the following four steps:

1) Mesh the substructure with a FEM tool, use generally valid guidlines for the choice of element type and size.
2) Define the interface points.
3) Export the model from FE environment by reducing its order with, e. g., the Guyan or Craig-Bampton method.
4) Execute a modal transformation and convert the reduced model to SID format.

V. Flexible models of a 6-axes industrial manipulator

A. *Lumped model with flexible joints*

Before establishing flexible link models, a verified lumped parameter model is described in the following. The model consists of seven rigid bodies and flexible joints with three-dimensional spring-damper pairs. Motors and gears are modeled as separate components with mass and interia. The advantage of the lumped model lies in its simplicity: With only few DOFs, the model describes the dynamic behavior of the robot accurately enough for many applications and is suitable for simulation and control purposes [14].

The rigid body parameters (mass, link length, center of mass position and inertia tensor), are known from the corresponding CAD model, while inertia values of the motors and gearboxes are obtained from the data sheets provided by the components' manufacturers. The lumped stiffness parameters do not represent the stiffness and damping of any individual physical component. They have been estimated based on measurements by using a frequency response technique as

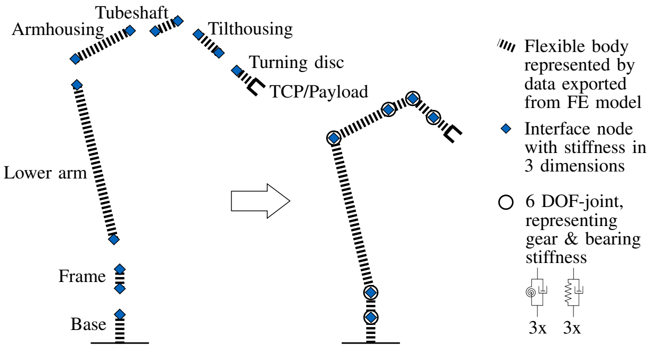


Fig. 3: Schematic drawing of the S-node Flex-Model.

described in [24] and [14]. The described model, which is fitted to and verified by measurements, serves as reference for the new Flex-Model (see following section), which aims to avoid the necessity of measurement data, i. e. the necessity of a real manipulator prototype.

B. Flexible link model (S-node Flex-Model)

The serial Single-Node Flexible-Link-Model (short "S-node Flex-Model") shows a structure, resembling to the lumped model. The main difference between the two models lies in the replacement of the lumped model's rigid bodies: The base and the six links of the manipulator are modeled as flexible bodies by including the reduced FE models, see Fig. 3. Applying the Guyan or the CB model order reduction, the so-called master nodes have to be defined. These nodes are selected as retained DOFs, i. e. as interface nodes. The links of the S-node Flex-Model" have two interface nodes each, where forces and torques can be applied in the multi-body simulation. Located at the joint positions, in the center of the bearings, the interface nodes are connected to the meshed surface of the solid part by rigid beam elements (see e. g. the lower arm in Fig. 4)

The idea of the modeling approach that underlies the S-node Flex-Model is to obtain a manipulator model by using only data that is available before the existence of a real prototype. Fitting parameter values to measurements, as it is done for the lumped model, is not desired. All data related to the manipulator's links (geometry, mass, stiffness, etc.) is obtained from the FE model. The lumped stiffness values of the flexible joints are gained from component datasheets and do not need to be estimated. It should be stressed that datasheets often contain average or minimum values. Furthermore, the stiffness characteristics of gears and bearings in the S-node Flex-Model are assumed to be linear. This simplification is necessary, because no measurement data is available during development to estimate the parameters of advanced (nonlinear) gear or bearing submodels.

A variant of the serial S-node Flex-Model was implemented, which takes into account the kinematic coupling of the wrist joints 4, 5, and 6.

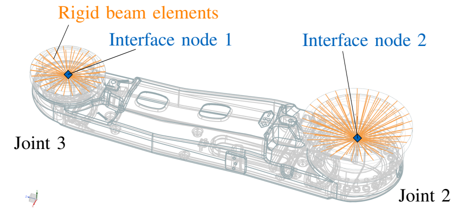


Fig. 4: Interface nodes of the lower arm.

VI. Model validation

The S-node Flex-Model is based on some simplifying assumptions regarding the flexible bodies and the model structure, and was created using purely data available during development. In order to validate that it still is a realistic and relevant model, some simulation experiments are presented in the following.

A. Dynamic behavior

The dynamic behavior of the model is validated by calculating the frequency response of the linearized system and visualizing it in Bode plots. Therefore, the manipulator is initialized in several different poses. Defining the motor torques as input to the system and the motor accelerations as output, results in a MIMO system of size 6×6.

The FRFs of the lumped model and the S-node Flex-Model with and without coupled wrist are qualitatively compared to the FRFs measured on a real manipulator. The 6×6 amplitude plots for an arbitrary robot pose are shown in Fig. 5. The columns refer to the input signals, i. e. the motor torques, and the rows refer to the motor accelerations (output signals). The FRF in column 3 and row 1, for example, represents the frequency response of the manipulator with a torque signal excited on motor 3 and the acceleration measured in motor 1. The enlarged plot of this FRF for the chosen manipulator pose is shown in Fig. 6. The FRFs are plotted for frequencies up to 100 Hz, because this range is of particular interest for the applications of a common medium size industrial manipulator. As expected, the lumped model mostly matches the measured FRF, since its parameters were estimated in order to fit these curves. The average absolute deviation between the 36 FRFs of model and measured curves is 5.5 dB for the range between 4 and 100 Hz. The dynamic behavior of the S-node Flex-Model agrees also well with the measurement, showing only an avg. abs. deviation of 4.5 dB. Here, the good match is not obvious, remembering that the model was built purely based on FE- and datasheet data. For some FRFs, for example the one shown in Fig. 6, the S-node Flex-Model matches the shape of the measured curve much better than the fitted lumped model. Depending on the pose of the manipulator, the differences between the S-node Flex-Model with and without coupled wrist become more noticeable. The behavior of a simple rigid body model with a linear spring-damper in the rotational direction of each joint is shown as reference. This "Rigid model" has an avg. abs. deviation of 9.8 dB to the measured FRFs.

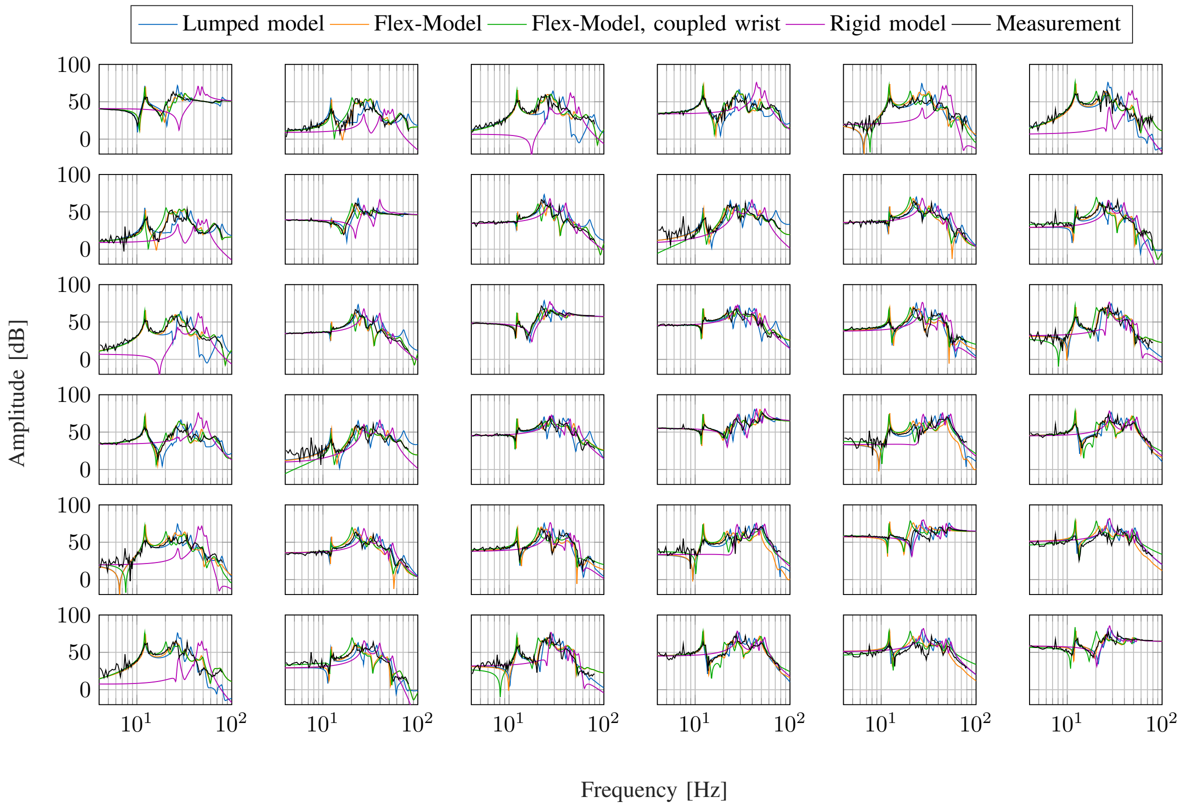


Fig. 5: Matrix of FRFs from motor torque (columns) to motor acceleration (rows) for one robot pose.

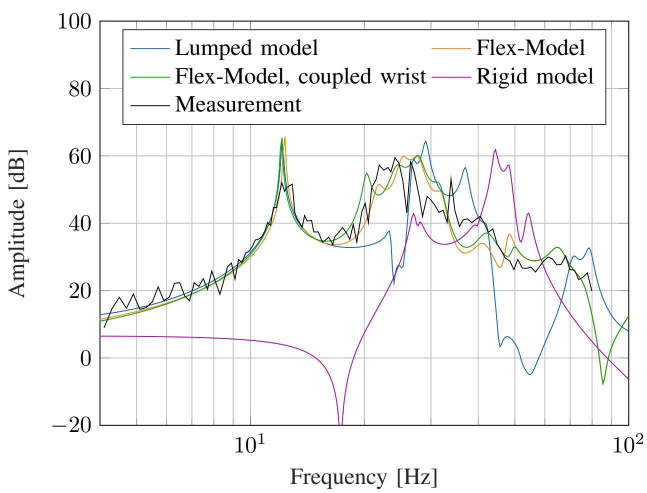


Fig. 6: FRF from motor torque of axis 3 to acceleration of motor 1 for one robot pose.

B. *Effects of the model order reduction technique*

Since the flexible bodies are the essential components of the S-node Flex-Model, the differences in the FRFs due to the chosen model order reduction technique are analyzed. The Guyan- and the CB reduction method are applied to reduce the flexible body description of the robot's links. For the CB-reduction, two different reductions are carried out including eigenfrequencies up to 500 Hz and up to 1000 Hz. The FRFs of the three model variants with differently reduced flexible links all match the measurement in a wide range and for frequencies lower than 100 Hz they do not differ. The model with Guyan-reduced flexible bodies deviates from the variants with CB-reduced SID for frequencies just higher than 100 Hz. The two models with the CB reduced flexible bodies differ from each other for frequencies over 300 Hz. Especially in the off-diagonal elements of the 6×6 matrix the model order reduction technique affects the shape of the curves. Since measurements were only available for low frequencies, it cannot be decided, which of the models variants is most accurate. However, the most common applications of a medium size industrial robot require to analyze the dynamic behavior only in the frequency range up to 100 Hz, where the curves lie on top of each other. This leads to the conclusion that the Guyan reduction method is sufficient to describe the dynamic behavior of the manipulator. Due to the smaller size of the SID-file, the Guyan reduction is preferred to the CB method. In this paper, the S-node Flex-Model is therefore configured with Guyan-reduced flexible bodies in order to keep system size and simulation time as small as possible.

C. *Path accuracy*

A test path for evaluating the positioning path accuracy is simulated with the multibody models in interaction with an experimental robot controller. The same controller was used for the simulated models and the real manipulator. The model-based control algorithm consists of a feedforward and a feedback loop, similar to the one described in [5]. The test path lies in the horizontal plane and is followed by the robot's tool center point (TCP) with a constant speed of 500 mm/s. A tool with nominal payload is mounted to the turning disc to create a more realistic test case. In this context the model accuracy refers to the maximum distance between the simulated and the measured TCP path. The coordinates of the real manipulator's TCP are measured using laser scanners, while the path of the model's TCP is recorded during the co-simulation of the Dymola model with the controller.

Some sections of the simulated path, as well as the ideal trajectory and the measured path of a real manipulator are shown in Fig. 7. The Flex-Models experience slightly more overshoot in the edges of the path than the lumped model and than the real robot (see Fig. 7a and 7c). This can be explained by a less stiff, or less damped behavior: A less damped behavior might be caused by the absence of friction in the gears and joints of the Flex-Model, while assumptions concerning the modeling of bearing stiffnesses within the flexible bodies may result in a less stiff behavior. Along the straight sections of the path, the S-node Flex-Model has the potential to cover the measured behavior better than the lumped model, see Fig. 7b.

The lumped model gives the best average model deviation along the complete test path, as shown in Fig. 7. The S-node Flex-Model gives 52 % higher, and the variant with coupled wrist 53 % higher average deviation, both relative to the lumped model average deviation. The better performance of the lumped model, compared to the Flex-Models, can be explained by the fact that its parameters were fitted to measurements, while the parameters of the Flex-Models were purely gained from development and datasheet data. The bearing stiffness values that are given in datasheets usually represent nominal values and thus not the actual, measured stiffness of the real component. Furthermore, the bearing stiffnesses were simplified to be linear and covered by an average value, while in reality, they have nonlinear characteristics, which depend on the load's magnitude and assumed point of application. Another explanation for the deviations of the Flex-Models' dynamic behavior bases on the choice of interface nodes for the flexible bodies. The straight forward approach of only one node between two bodies may not be sufficient to describe the dynamic robot behavior accurately, since simplifying assumptions need to be made in order to create interface points within a FE-substructure. The positions of the interface nodes was assumed to be in the center of each bearing between two links. For a bearing arrangement, consisting of two angular contact ball bearings, for example, the interface node was chosen in the center of their shared axis of rotation, assuming both bearing partners to be loaded equally.

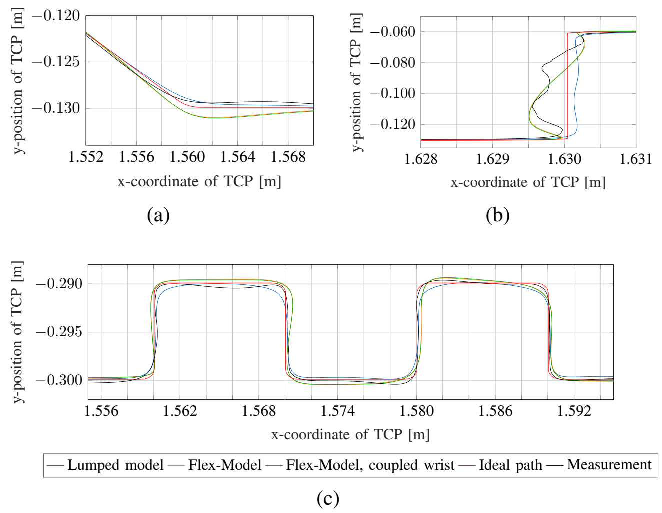


Fig. 7: Enlarged sections of the test path; simulated and measured TCP positions with an experimental path controller.

VII. CONCLUSIONS

Motivated by the current trend to develop weight- and cost-minimized industrial robots, the design of the structure needs to be optimized, and a large amount of applied research is needed in order to improve the dynamic performance. This paper treated the inclusion of flexible bodies in the multibody model of a 6-axis manipulator. One of the main advantages of this approach is the avoidance of a parameter estimation process. The information, which describes the flexible behavior of each substructure, is exported from the corresponding FE model by using model order reduction techniques. The so-called S-node Flex-Model, which is based purely on the corresponding FE model and datasheet data, was presented and compared with a lumped parameter model with rigid bodies and parametrized flexible joints. The dynamic model behavior of the S-node Flex-Model was validated by comparing the FRFs to measurement data of a real manipulator and a convincing match of the amplitude curves was demonstrated. The Guyan and Craig-Bampton method were applied to reduce the FE models of the manipulator links and it was shown that the dynamic behavior differs only for high frequencies, which are less relevant for many applications. For further validation, the multibody models were co-simulated with an experimental robot controller, and a test path was executed. The model accuracy, which refers to the ability of a model to replicate the TCP movement of the real robot, was estimated and showed a convincing result for the presented Flex-Models.

In order to make the manipulator control benefit from a more accurate robot model, which takes into account flexible body dynamics, the Flex-Model could be directly used in the controller algorithm. Another approach is using the Flex-Model to fit the parameters of a lumped model, which is commonly used for control. This gives the advantage that no real prototype is needed for the parameter fit process and the controller can be tuned in an early phase of the manipulator development.

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