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Efficient and precise sensor fusion for non-linear systems with out-of-sequence measurements by example of mobile robotics

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Abstract— **For most applications in mobile robotics, precise state estimation is essential. Typically, the state estimation is based on the fusion of data from different sensors. In practice, these sensors differ in their characteristics and measurements are available to the sensor fusion algorithm only with delay. Based on a brief survey of sensor fusion approaches that consider delayed and out-of-sequence availability of measurements, suitable approaches for applications in mobile robotics are identified. In a consumer robot use-case, experiments show that the estimation is biased if delayed availability of measurements is not considered appropriately. However, if delays are considered in the fusion process, the estimation bias is reduced to almost zero and in consequence, the estimation performance is distinctly improved. Two computational favorable approximative methods are described and provide almost the same accuracy as – theoretically optimal – brute-force filter recalculation at much lower and well-distributed computational costs.**

I. INTRODUCTION

Fusing data from different sensors for state estimation is a very common task in robotics. Motion planning and control in mobile robotics require precise and timely estimations of the robot's pose. Ideally, the overall latency for pose estimation is bounded to few milliseconds. In motion planning and control for manipulation, the real-time requirements may be even more demanding.

In practical systems, the latency of sensor data depends on the measurement principles, sensor-internal preprocessing and communication technologies between the sensors and the main computing device. Many sensors run internal filter and fusion algorithms so that the overall sensor fusion is a staged and distributed process. Examples for such sensors are GPS receivers and most IMUs. In this context, SLAM may be also considered as a very advanced localization "sensor".

Typically, to meet the real-time requirements by motion planning and control best possible, a greedy approach is used. That is, the sensor fusion integrates all available measurements as quickly as possible – knowing full well that there might be delayed measurements arriving later. For filter-based state estimation, there are two trivial approaches to handle delayed measurements: First, the delays may be disregarded at all, considering the data as being just measured, which results in a bias in the estimated pose. Second, the sensor fusion algorithm maintains a time-bounded history of the last measurements, states, and related filter information.

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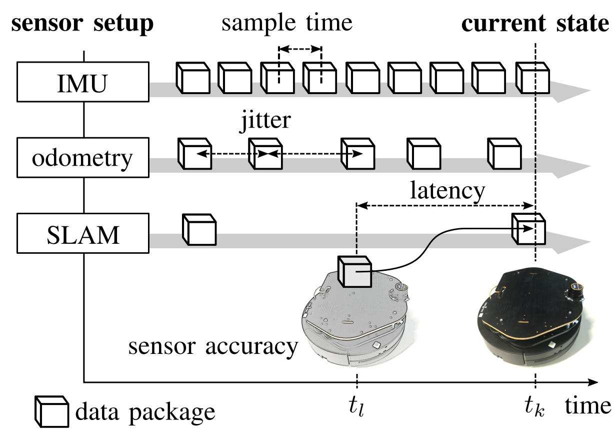


Fig. 1: Typical applications in mobile robotics require the fusion of sensor data from different sources. These sources differ greatly in their characteristics, including sampling times and delays.

When the delayed measurement arrives, all measurements during the delay period are reprocessed to update the filter state. This recalculation results in a higher accuracy but also in higher computational effort. The second approach requires that the sensor data is timestamped, ideally at acquisition time by the clock-synchronized sensor itself.

In different application domains, a number of approaches in-between the two trivial ones – i.e. in-between disregarding the delays and recalculation – have been developed. These approaches are often tailored to certain use-cases or system characteristics. Some of these approaches can be applied to general non-linear systems and various delay characteristics while others expect certain delay characteristics or assume a linear system. Altogether, these approaches allow a trade-off between accuracy and computational cost, both, in terms of computing time and memory consumption.

In this paper, we present a theoretical and experimental analysis of these approaches in the domain of mobile robotics. In detail, our contributions are as follows:

- Brief survey of sensor fusion approaches for delayed and out-of-sequence measurements.
- Taxonomy and conceptual framework for comparing these approaches on a feature-level.
- Report on the implementation of two promising approaches for mobile robotics in consumer use-cases.
- Experimental results from these two approaches and comparison with the optimal recalculation scheme.

The remainder of the paper is organized as follows: Section II introduces the problem statement and taxonomy to review and characterize properties of approaches to handle delayed measurements. Section III provides a survey of sensor fusion approaches for delayed and out-of-sequence measurements. Finally, Section IV describes conducted experiments and results to assess the performance of selected approaches in an indoor consumer robotics use-case.

II. PROBLEM STATEMENT AND DEFINITIONS

The evolution of the system state x from timestep $k-1$ to timestep k is modeled by the system equation

$$x_k = f(x_{k-1}, u_{k-1}, w_k) \quad (1)$$

with the system input u and the system noise w. The subscript denotes the timestep. Timesteps are not required to be equidistant. Properties of the system state at timestep l are captured by a measurement y_l^k according to the measurement equation

$$y_l^k = h(x_l, v_l) \quad (2)$$

with measurement noise v_l.

The measurement y_l^k is available to the sensor fusion algorithm at some timestep $k \geq l$ (see Fig. 1) with the delay definitions as follows: If $k = l$ the measurement is *non-delayed*, if $k - l = 1$ the measurement is called *single-step delayed*, otherwise *multi-step delayed*. If a measurement is delayed and all other measurements in the delay-period are non-delayed or no other measurements are acquired at all, the measurement is called *one at a time delayed*. If delay periods of different measurements overlap, the measurements are *concurrently delayed*. Typically, delays entail that measurements are not available in their order of acquisition; such measurements are called *out-of-sequence*.

The existence of a measurement and its acquisition time is either known in advance (e.g., in triggered systems) or becomes known only at the time the timestamped measurement becomes available. The timespan between measurement acquisition and its availability to the sensor fusion algorithm is called *latency*. The latency is either deterministic or non-deterministic.

In mobile robotics, state estimation typically relies on multiple sensors that differ widely in their characteristics, including varying sampling time and delay as illustrated in Fig. 1. Hence, a sensor fusion system in mobile robotics must be able to handle multiple concurrent out-of-sequence measurements with multi-step delays in order to fulfill crucial tasks like fine positioning and obstacle avoidance. Typically, delays include non-deterministic jitter and acquisition times are not known until the timestamped measurement is available. Only in special cases some of these requirements can be mitigated: For example, triggered measurement systems with real-time capable communication may provide known latencies and acquisition times. Acquisition times might also be estimated by monitoring the system as proposed in [1].

Filter-based approaches to handle delayed measurements are generally built upon underlying algorithms that are originally formulated for non-delayed measurements. Depending on the system and measurement characteristics, different state estimation algorithms – e.g., linear and non-linear Kalman filters or particle filters – are used to estimate the unknown system state based on available measurements.

In order to compare approaches to consider delayed availability of measurements in the fusion process, the following criteria are proposed:

- An approach is considered *optimal* if it provides equal results as another hypothetical system with the same underlying algorithm that processes a set of non-delayed (but otherwise identical) measurements. Hence, in this scope, optimality is defined in terms of optimally compensating delayed availability of measurements.
- The *general applicability* of an approach indicates the extent to which it can be used for different systems and boundary conditions. The general applicability is limited if an approach is only suitable for linear systems, is only capable of handling certain delay characteristics, or requires some knowledge about the measurement already before its availability.
- A generally applicable approach is *agnostic* if underlying estimation algorithms – e.g., extended Kalman filter (EKF) or unscented Kalman filter (UKF) – can be interchanged without further adaptation.
- When limited hardware is used and real-time requirements have to be met, important selection criteria are *computational costs and memory consumption*. The selection of a suitable approach is generally a trade-off between computational costs, memory consumption, and accuracy.

III. STATE OF THE ART

Based on these criteria, Table I provides a characterization of methods to handle delayed measurements. We classify these methods into three groups as described in the following:

1. Disregarding the delays. The methods of this group assume that the latency is sufficiently small and that unconsidered delays do not cause any intolerable error. In consequence, the delay is disregarded and the measurements are processed as if they were in correct chronological order and not delayed. This approach is simple to implement and does not add computational costs. However, disregarding the delay causes systematic errors and is the most improper method [1], [2]. The accuracy of this method is mainly improved by decreasing the delay itself, e.g., by using powerful hardware and efficient postprocessing of measurements [3]. To enforce that the assumption of negligible latency holds, delayed measurements might be rejected. Instead of rejecting all delayed measurements, adaptive rejection of delayed measurements is proposed in [4], [5], [6]. However, rejecting measurements causes a loss of information and the filter accuracy may decrease or no proper estimation may be provided at all [7], [8], [9].

2. Brute-force methods. The methods of this group recalculate the entire filter, which requires to save some state

TABLE I: Characterization of methods to handle delayed measurements

	optimal * for linear systems	non-linear system	delay characteristics	agnostic	requirements at acquisition time	storage including related statistics	computation costs † distributed evenly over delay period
disregard delay	-	✓	no restrictions	✓	-	-	very low
discard delayed measurements	-	✓	no restrictions	✓	-	-	very low
A1 / B1 / C1	✓* / - / -	-	single-step	-	-	-	medium / low / low
Al1 / Bl1	-	-	no restrictions	-	-	estimates	medium / low
single state augmentation	-	✓	one at a time	-	timestamp	-	medium†
multiple / full state augmentation	-	✓	no restrictions	-	-	estimates (implicit)	very high†
Alexander's method	-	✓	one at a time	-	meas. covariance	-	medium†
Alexander's method (2-filter)	✓*	✓	one at a time	-	meas. covariance	-	high†
Larsens's method	-	✓	no restrictions	-	-	estimates	medium
CI1 / CI2	-	✓	no restrictions	-	-	estimates	low
brute-force recalculation	✓	✓	no restrictions	✓	-	measurements, estimates	high

estimates and all measurements during the delay period, including related statistics. Most implementations employ the following scheme: When a delayed measurement arrives, it

- sorts this measurement into the history of measurements according to the acquisition timestamp,
- resets the filter to the latest state before this timestamp, and then
- recalculates the state and further filter information iteratively by means of the history.

The lengths of the delay periods determine the computational costs. These costs are not well distributed, but rather arise on single occasions when recalculation becomes necessary. Therefore, filter recalculation may be unsuitable for real-time applications. In return, filter recalculation yields optimal results [1], [8], [9], [10], [11]. In the domain of mobile robotics, the filter recalculation scheme is implemented in *robot_localization* package with an EKF and an UKF for the Robot Operating System (ROS) [12]. Lynen et al. describe an implementation of this scheme for sensor fusion for micro aerial vehicles in [13].

3. Direct fusion. The methods of this group consider delayed availability and directly fuse the delayed measurement with the current state estimate without the need for reprocessing former measurements. These approaches fall in-between the two former ones: They aim to provide increased estimation accuracy compared to disregarding the delays at reduced computational costs compared to the brute-force recalculation scheme. These methods are essentially based on two principles:

1) It is considered that the measurement captures properties of the state at acquisition time and not of the current estimate. Hence, the *corrected measurement innovation* (CI) $\tilde{y} = y_l^k - h(\hat{x})$ is based on an estimate $\hat{x} = \hat{x}_l$ of the state at the time of the measurement acquisition rather than on the current estimate $\hat{x}_k$ at measurement availability.
2) The (Kalman) gain takes into account that the current state estimate and the measurement are related to different times. If the measurement is non-delayed, hence $k = l$, an optimal gain can be derived in closed-form. For example, for linear systems, this leads to the ordinary Kalman filter equations. In the case of delayed measurements, hence $k > l$, the key aspect is that the optimal gain includes terms that originate from the cross-covariance between the state estimates at different timesteps [1], [14]. Therefore, research on this topic focuses on different optimal and approximative methods that include – either implicitly or explicitly – this cross-covariance in the derivation of the gain.

Bar-Shalom provides several algorithms for linear systems with delayed measurements: For single-step delayed measurements, an inverse system model can be used to obtain both the needed statistics to compute the gain and the state estimate at measurement time [15]. The so-called algorithm A1 [15] provides an optimal implementation of this method, whereas the algorithms B1 [15] and C1 [15] are simplifications thereof and provide suboptimal solutions at reduced computational costs. The algorithms Al1 [16] and Bl1 [16] expand the aforementioned algorithms to linear systems with multi-step delayed measurements by condensing multiple measurements into one single virtual measurement. This requires to store the preceding estimates. Using this single virtual measurement, the problem is transformed into a single-step delay problem and hence the algorithms A1 and B1 can be applied. Although the use of algorithms A1, B1, Al1, Bl1 for special non-linear systems has been discussed in [6], [17], their applicability to general non-linear systems with multi-step delayed measurements is limited, since it is required that the influence of multiple measurements can be adequately condensed into one single value and an inverse system model can be deduced for the backward prediction.

For general non-linear systems, the cross-covariance between the state estimate at acquisition and current time can be obtained by augmenting the system state and its covariance. Therefore, the use of state augmentation to compute the gain is proposed for various linear and non-linear sensor-fusion algorithms [1], [18], [19]. Yet, this approach requires that the existence of the measurement is already known to the fusion algorithm at acquisition time. If this requirement is not met or multiple measurements are delayed concurrently, the state must be augmented several times or fully, i.e. for each timestep within a reasonable timeframe. Each augmentation increases the dimension of the state vector. Since the dimension of the state vector determines the computational effort in most estimation filters, multiple state augmentation imposes high computational costs for systems with high dimensional states, unknown acquisition times and multiple concurrent delays. In consequence, the computational burden is most likely higher than for complete filter recalculation and often infeasible in practice [2], [19].

To avoid the need for explicitly considering the cross-covariance of the state estimate at different timesteps, Alexander's method [20] updates the state estimates' covariances at measurement acquisition time as if the measurement itself already would have been available. The measurement itself is later incorporated into the estimate. A correction term accounts for the evolution of the system and ensures optimality – at least for linear systems – after the measurement has been incorporated. Optimality during the delay period can be achieved by simultaneously using a second filter instance whose covariance is not updated at acquisition time [10]. Yet, this approach requires also that the existence of the measurement and its covariance is already known to the fusion algorithm at acquisition time.

Since this requirement can often not be met in practical applications, Larsen's method [10] builds on Alexander's method and also utilizes a correction term to incorporate the delayed measurement but omits to update the state estimates' covariances at acquisition time. Although Alexander's and Larsen's methods are originally derived for linear systems only, they can be expanded for the use with non-linear systems, e.g., in combination with the EKF or the class of sigma-point Kalman filter (SPKF), including UKF and central difference Kalman filter (CDKF) [1].

Based on the assumption that the change of estimation error in the delay period is negligible, computational undemanding fusion methods can be derived which can also be regarded as approximations to the aforementioned direct fusion methods [2]. These methods do consider that the measurement captures properties of the state at acquisition time and hence the measurement innovation is based on an estimate of the state $\hat{x}_l$ at the time of the measurement acquisition. As opposed to the other direct fusion methods, it is disregarded that the current state estimate and the measurement are related to different times for the derivation of the gain. In consequence, the derivation of the gain is simplified and the computational burden is similar to the processing of a non-delayed measurement. This approximation proposed by [1], [2] can be implemented in different ways. Two different implementations – namely CI1 based on the current state estimate statistics and CI2 based on the statistics at acquisition time – are described in the following.

IV. EXPERIMENTS

Despite disregarding the delay and discarding delayed measurements, Larsen's method, the approaches CI1 and CI2, full state augmentation and filter recalculation are applicable to non-linear systems and meet the typical requirements in mobile robotics as stated in Section II. In our experiments, we examine the performance of computational favorable approaches CI1 and CI2. For experimental evaluation, the approaches are implemented based on the EKF and the square-root CDKF as presented in [1]. An ordinary EKF and CDKF implementation – disregarding delayed availability – serves as baseline. Filter recalculation is included in the comparison as theoretically optimal approach.

The approaches CI1 and CI2 are implemented as follows: Maintain a time-bounded history of the previous state estimates (and related covariances for CI2). When a delayed measurement y_l^k with measurement covariance R_l arrives,

- look up the related state estimate[1] $\hat{x}_l$ (and related covariance $P_{x,l}$ for CI2).
- For CI1 compute the gain based on the current prior state estimate $\hat{x}_k^-$ and its covariance $P_{x,k}^-$. For example, for the EKF-based implementation this is

$$H = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_k^-}, \tag{3}$$

$$K = P_{x,k}^- H^T (H P_{x,k}^- H^T + R_l)^{-1}. \tag{4}$$

  For CI2 compute the gain based on the related estimate $\hat{x}_l$ and its covariance $P_{x,l}$. For example, for the EKF-based implementation this is

$$H = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_l}, \tag{5}$$

$$K = P_{x,l} H^T (H P_{x,l} H^T + R_l)^{-1}. \tag{6}$$

- Update the current prior state estimate $\hat{x}_k^-$ and its covariance $P_{x,k}^-$ with the delayed measurement using the related estimate $\hat{x}_l$, the gain K as defined above and the update equations of the underlying algorithm. For example, for the EKF-based implementation the state update is

$$\hat{x}_k^+ = \hat{x}_k^- + K(y_l^k - h(\hat{x}_l)), \tag{7}$$

  and the covariance update is

$$P_{x,k}^+ = (I - KH) P_{x,k}^- (I - KH)^T + K R_l K^T. \tag{8}$$

Analogous to the EKF-based implementation, the CI1 and CI2 approach can be applied to different underlying algorithms. For the CDKF implementation in our experiments, the sigma-point approach[2] [1] is used to obtain the gain and

[1] In practical applications with variable timesteps, an exact match of the measurement acquisition time and the stored estimates is unlikely. Therefore, we use the closest available estimate before the measurement acquisition and propagate it forward by using the time-update equations of the underlying algorithm in order to obtain the related estimate $\hat{x}_l$.

[2] This necessitates maintaining the covariance in the history also for CI1.

TABLE II: Properties of experiment trajectories

experiment	length [m]	duration [s]	$\|v\|$ [m/s]		$\|\dot{\varphi}\|$ [°/s]	
			mean	std-dev	mean	std-dev
(R1a)	23.07	57	0.40	0.11	24.74	15.96
(R1b)	24.08	59	0.41	0.14	24.51	17.32
(R1c)	23.63	56	0.42	0.14	25.26	18.67
(R2)	17.65	80	0.22	0.08	23.27	13.77
(R3)	39.18	87	0.45	0.22	32.42	37.27
(R4)	30.55	154	0.20	0.12	8.43	11.85

to propagate the estimate and covariance through the system and measurement equation.

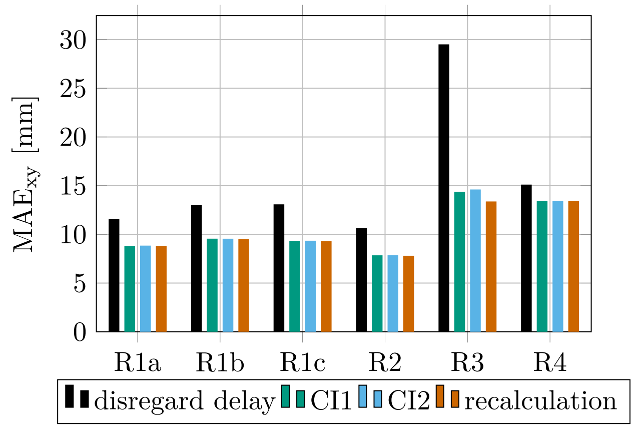
Experiments are conducted in a flat environment. The system is modeled as 2D constant-translational-acceleration and constant-angular-velocity model. Measurement parameters, i.e. gyroscope and accelerometer bias, and odometry parameters, i.e. wheel scale-factors and effective wheel-track are included into the state vector and jointly estimated together with the robot's pose and associated derivatives. The estimation filters are executed with a fixed frequency of 100 Hz. If more than one measurement becomes available between two consecutive filter execution steps, the measurements are processed in chronological order.

A total of six experiments are conducted and cover different dynamic scenarios. Experiment properties are listed in Table II. To evaluate the reproducibility of results, three experiments – (R1a), (R1b), (R1c) – repeat a similar twofold 8-shaped trajectory. The experiment (R2) follows a threefold 8-shaped trajectory at reduced linear velocity v and angular velocity $\dot{\varphi}$. The experiment (R3) follows a trajectory including sections with linear movement only, angular movement only, and combined linear and angular movement of varying velocities. The experiment (R4) follows a rectangular shape that is first performed counter-clockwise and then clockwise, including short periods of zero movements.

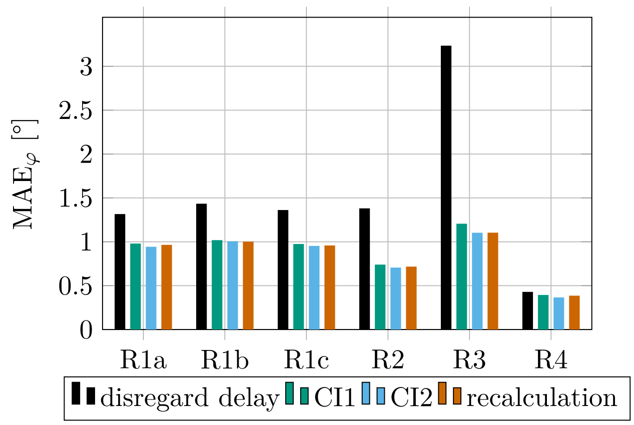
Yujin Robot's mobile robot base Kobuki[3] is used as experiment platform. The sensor setup consists of two native wheel encoders (timestamped at the time of ROS-message creation by the ROS wrapper for the Kobuki driver; the on-board computer running ROS is time-synchronized via NTP), an Xsens MTi-300-2A8G4 IMU (timestamped at acquisition time by Xsens MTi-300-2A8G4; the device is time-synchronized via GPS) and global pose information provided by SLAM based on Hokuyo UTM-30LX-EW laser scanning rangefinder measurements (timestamped at acquisition time by Hokuyo UTM-30LX-EW; the device is time-synchronized via the Hokuyo driver synchronization routine). Measurements are recorded on the robot's on-board computer.

IMU measurements are available with a fixed sampling time of 10 ms and a typical delay below 0.5 ms and a maximum delay of up to 10 ms; wheel encoder measurements are available with a sampling time between 15 ms and 65 ms and typical delay below 0.5 ms and a maximum delay of up to 11 ms; global pose information from SLAM is available with

[3]http://kobuki.yujinrobot.com/



(a) Mean absolute position error $\mathrm{MAE_{xy}}$



(b) Mean absolute orientation error MAE_{φ}

Fig. 2: Localization error (EKF-based implementations)

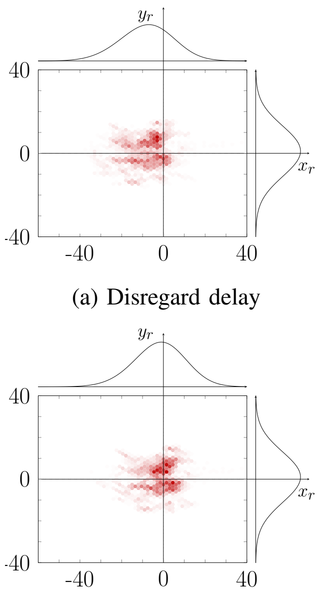
a sampling time between 30 ms and 340 ms. The delay of the global pose information is mainly distributed between 10 ms and 60 ms with a median delay in all experiments ranging from 20 ms to 25 ms. However, some pose data have a considerably higher delay. In (R1a-c), (R2), (R4) measurements include outliers with delays up to 200 ms. The experiment (R3) contains considerably more global pose data with delays above 35 ms than the other experiments, also including some outliers with delays over 200 ms.

Ground truth data is recorded at 120 Hz with a NaturalPoint OptiTrack external motion capture system.[4]

In our experiments, we assess the accuracy of the state estimation in terms of the mean absolute position error $\mathrm{MAE_{xy}}$ and the mean absolute orientation error MAE_{φ} of the estimated robot pose. The localization accuracy achieved by the approaches based on the EKF implementation is visualized in Fig. 2. In the conducted experiments, EKF and CDKF based implementations of the approaches perform almost equally. Therefore, the results of the CDKF-based implementation are only listed in Table III.

Our experiments show that the estimation performance in terms of localization accuracy is considerably increased if the

[4]Mean 3D error in calibration between 0.369 mm and 0.410 mm (overall reprojection).



(a) Disregard delay (b) Recalculation

(c) CI1 (d) CI2

Fig. 3: Distribution of position estimates in the robot coordinate system. Experiment (R1a), EKF-based implementations, direction of movement: x_r, values in mm.

delayed availability of measurements is taken into account. Furthermore, they indicate that the suggested approaches CI1 and CI2 provide improvement of estimation accuracy similar to a complete filter recalculation for the used test platform. This finding is backed up by simulation experiments[5] with simulated delay ranging between $20\,\mathrm{ms}$ and $135\,\mathrm{ms}$. Under similar conditions, i.e. (R1a-c), the experiments also reproduce similar results as shown in Fig. 2 and Table III.

By considering the delayed availability of measurements, the mean absolute position error $\mathrm{MAE_{xy}}$ is decreased roughly by around $25\,\%$ in the experiments (R1a-c) and (R2). In (R3), global position data is affected by larger delays, in consequence, the decrease of $\mathrm{MAE_{xy}}$ is more pronounced. The less pronounced decrease in position error in (R4) is likely to be caused by the fact that the trajectory in (R4) includes idle times in the robot's movements and since the robot's position is not changing during these idle times, the influence of delays is less severe. Concerning the mean orientation error, qualitatively the same improvement by considering the delayed availability is observed in our experiments. The overall low orientation error in (R4) results from the rectangular trajectory with mostly straight driving – also yielding low mean angular velocity (see Table II).

The loss of accuracy in case of unaccounted measurement delay is basically caused by a biased estimation. In all experiments, but also in simulation, the estimate is noticeably biased if the delay is disregarded in the ordinary EKF and CDKF implementation. This is exemplarily visualized for experiment (R1a) in Fig. 3a by plotting the density of position estimates in the robot's coordinate system. For

[5]The simulation is based on a mobile robot platform with properties similar to the Kobuki robot base and comparable sensor setup.

TABLE III: Experiment results: Localization accuracy

(a) Mean absolute position error $\mathrm{MAE_{xy}}$ in mm

	R1a	R1b	R1c	R2	R3	R4
disr. delay (EKF)	11.591	12.990	13.082	10.636	29.510	15.411
disr. delay (CDKF)	11.608	12.989	13.083	10.621	29.502	15.411
CI1 (EKF)	8.817	9.556	9.338	7.848	14.375	13.424
CI1 (CDKF)	8.840	9.551	9.334	7.823	14.337	13.421
CI2 (EKF)	8.844	9.552	9.346	7.866	14.609	13.431
CI2 (CDKF)	8.875	9.546	9.342	7.841	14.558	13.428
recalculate (EKF)	8.822	9.522	9.313	7.803	13.379	13.421
recalculate (CDKF)	8.844	9.519	9.310	7.779	13.363	13.421

(b) Mean absolute orientation error MAE_{φ} in $^\circ$

	R1a	R1b	R1c	R2	R3	R4
disr. delay (EKF)	1.316	1.434	1.362	1.381	3.235	0.429
disr. delay (CDKF)	1.315	1.430	1.358	1.375	3.232	0.428
CI1 (EKF)	0.980	1.019	0.975	0.740	1.206	0.394
CI1 (CDKF)	0.979	1.015	0.971	0.736	1.206	0.397
CI2 (EKF)	0.943	1.006	0.953	0.706	1.103	0.366
CI2 (CDKF)	0.947	1.014	0.949	0.702	1.102	0.369
recalculate (EKF)	0.965	1.002	0.958	0.717	1.104	0.386
recalculate (CDKF)	0.965	0.998	0.955	0.714	1.102	0.388

an unbiased estimation it is expected that the estimates are centered around the origin. However, when delays are disregarded, the position estimates systematically lag behind in the direction of movement x_r. In contrast, when delays are considered in the fusion process either by recalculating the filter (Fig. 3b) or with the approaches CI1 (Fig. 3c) and CI2 (Fig. 3d), the bias is reduced to almost zero.

V. CONCLUSION

The main findings in our experiments can be summarized as follows:

- The estimation is biased if delayed measurement availability is disregarded in the sensor fusion process.
- However, the estimation bias is reduced to almost zero if delays are appropriately considered. In consequence, the estimation accuracy is improved.
- Especially, the computational favorable approaches CI1 and CI2 provide almost the same improvement of accuracy as brute-force recalculation but with much lower and well distributed computational costs. This gives much smaller bounds for the computing time compared to recalculation and allows the implementation on small computing platforms.

In conclusion, the approaches CI1 and CI2 provide a promising solution to improve estimation accuracy when computational costs are a critical factor and real-time requirements must be met. In mobile robotics, this enables precise state estimation in the presence of delayed and out-of-sequence measurements and thus improved motion planning, control, and obstacle avoidance can be achieved.

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