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Design of Spatial Admittance for Force-Guided Assembly of Polyhedral Parts in Single Point Frictional Contact

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Abstract— **This paper identifies conditions for designing the appropriate spatial admittance to achieve reliable force-guided assembly of polyhedral parts for cases in which a single frictional contact occurs between the two parts. This work is an extension of previous work in which frictionless contact was considered. This paper presents a way to characterize friction without solving a set of complicated non-linear equations. We show that, by modifying the error reduction function and evaluating the function bounds associated with friction, the procedures developed for frictionless contact apply to the frictional cases. Thus, for bounded misalignments, if an admittance satisfies the misalignment-reducing conditions at a finite number of contact configurations, then the admittance will also satisfy the conditions at all intermediate configurations for any value of friction less than the specified upper bound.**

I. INTRODUCTION

Robotic assembly requires physical interaction with its environment. For effective use in tasks, robots should regulate the contact force and ultimately provide accurate part positioning. The force regulation and motion response of a robot are characterized by its mechanical admittance. If properly designed, an admittance provides error-reducing motion for all bounded misalignments within a given contact state.

Although an error-reducing motion response from contact is desired at all bounded misalignments, it is impossible to separately evaluate the infinite number of possible configurations that may occur. Thus, it is necessary to develop a set of sufficient conditions to be imposed on the admittance at a finite number of configurations to ensure that the motion response to contact is error-reducing. Once established, the conditions can be used as testable conditions useful in the search for an appropriate admittance matrix.

This paper presents conditions used to design an appropriate spatial manipulator admittance for force-guided assembly of two polyhedral objects when contact is frictional and is restricted to cases of single point contact. Here, a simple, general linear admittance control law [1] is used. For spatial applications, this type of admittance has the form:

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{A}\mathbf{w} \tag{1}$$

where $\mathbf{v}_0$ is the nominal twist (a 6-vector), $\mathbf{w}$ is the contact wrench (force and torque) measured in the body frame (a 6-vector), $\mathbf{A}$ is the admittance matrix (a 6×6 matrix), and $\mathbf{v}$ is the motion of the body.

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A. Related Work

Force guided assembly has been investigated in previous work [2, 3, 4, 5, 6] often in application to the peg-in-hole problem [7, 8, 9].

Many researchers have addressed the design of admittance for force-guidance [10, 11, 12, 13]. None of the general approaches, however, provides a proof that the admittance selected will, in fact, be reliable for all possible configurations. In recent work, more complicated admittance control has been investigated for human-robot interaction [14, 15].

In the force-assembly [1] approach used here, the control law is designed so that, at each possible part misalignment, the contact force always leads to a motion that instantaneously reduces the existing misalignment. In previous closely related work [16], sufficient conditions for admittance selection for *planar* assembly with friction were presented. More recently, admittance selection conditions for *frictionless* assembly of spatial polyhedral parts in single principal contact (PC) [17] and in two single-point PCs [18] were presented.

This work is an extension of our previous work on frictionless contact [17] to frictional cases. When friction is included, the problem for spatial cases is significantly more difficult. The error-reduction conditions require knowledge of the direction of the force of contact between the parts. It is known [19], however, that for a given compliance/admittance, the friction force and response motion are coupled and therefore must be solved simultaneously using a set of non-linear equations. Since the admittance is to be selected (and therefore unspecified), it is impossible to obtain the direction of friction using geometry alone (as in the frictionless case). Here, a new way of accounting for friction without solving the complicated equations is developed.

B. Approach

Similar to related work [16, 17, 18], here we consider a *measure* of error based on the Euclidean distance between an arbitrarily chosen single (fixed) point on the held body and its location when properly positioned.

Using this point-based measure of misalignment, misalignment reduction can be expressed as a function. If $\mathbf{d}$ (a 6-vector for spatial motion) is the line vector from the selected point at its properly mated position to its current position, then, for error reducing motion, the condition is:

$$\mathbf{d}^T\mathbf{v} = \mathbf{d}^T(\mathbf{v}_0 + \mathbf{A}\mathbf{w}) < 0. \tag{2}$$

Since force-assembly requires that misalignment is reduced at each possible misalignment, this condition must be satisfied for all misalignments in the range considered.

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Here, for contact friction, Coulomb's model is used. Thus for single point contact, using the reciprocal condition ($\mathbf{v}^T\mathbf{w}_n = 0$) [20] and the process used in [16], the error-reduction condition can be equivalently expressed as:

$$F_{1p} = \mathbf{d}^T (\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\, (\mathbf{w}_n + \mu \mathbf{w}_t) < 0 \qquad (3)$$

where μ is the coefficient of friction between the contact parts and where $\mathbf{w}_n$ and $\mathbf{w}_t$ are unit wrenches associated with normal and frictional contact forces respectively and have the form:

$$\mathbf{w}_n = \begin{bmatrix} \mathbf{n} \\ \mathbf{r} \times \mathbf{n} \end{bmatrix}, \quad \mathbf{w}_t = \begin{bmatrix} \mathbf{t} \\ \mathbf{r} \times \mathbf{t} \end{bmatrix} \qquad (4)$$

where $\mathbf{r}$ is the position vector from the origin of the coordinate frame to the point of contact C.

When compared to the error reduction function in frictionless cases [17], the error reduction function in (3) has the same form except that the contact wrench now contains the frictional component $\mu\mathbf{w_t}$. Thus, as in the frictionless contact cases [17] , the normal wrench $\mathbf{w}_n$ and error-measure vector $\mathbf{d}$ are determined by geometry alone and are expressed as functions of part configurations. When friction is included, the body motion and the friction force are coupled in more complicated nonlinear equations. The direction of the contact force, which is needed to determine the motion of the held body, also depends on the motion of the body. To obtain a set of admittance selection conditions without calculating the direction of friction, we assume that the conditions are satisfied for all possible directions of friction in the tangent plane. We show that, by evaluating the bounds of functions involving $\mathbf{w}_t$, the error-reduction condition for different types of PCs can be expressed in the same form as the corresponding frictionless case presented in [17]. Thus a set of sufficient conditions for error-reducing motion can be obtained using the same procedure used in [17]. Here, all friction coefficients within the range $[0, \mu_M]$ are considered. Since the design conditions are developed for single point contact, the identified conditions are only valid locally for the contact state.

C. Overview

In this paper, sufficient conditions for an admittance to ensure force-guided assembly despite friction are established for each of the PCs in which single point contact occurs. Section II identifies the coordinates used to describe configuration variation for each contact state. Sufficient conditions for error reduction for each PC are derived in Sections III-V. These conditions show that an admittance matrix that satisfies the error reduction conditions at the boundaries of a set of contact configurations also satisfies the error-reduction conditions at all intermediate configurations and all values of friction less than that specified. A discussion and a brief summary are presented in Section VI.

II. CONFIGURATION DESCRIPTION

Polyhedral bodies in single-point contact have three types of stable principal contacts [21]: "face-vertex" ($\{f\text{-}v\}$) contact, "vertex-face" ($\{v\text{-}f\}$) contact, and "edge-edge cross" ($\{e\text{-}e\}_c$) contact as shown in Fig. 1. In [17], the sets of coordinates used to describe configuration variation for each contact state are presented. The descriptions apply to frictional contact PCs and are reviewed below.

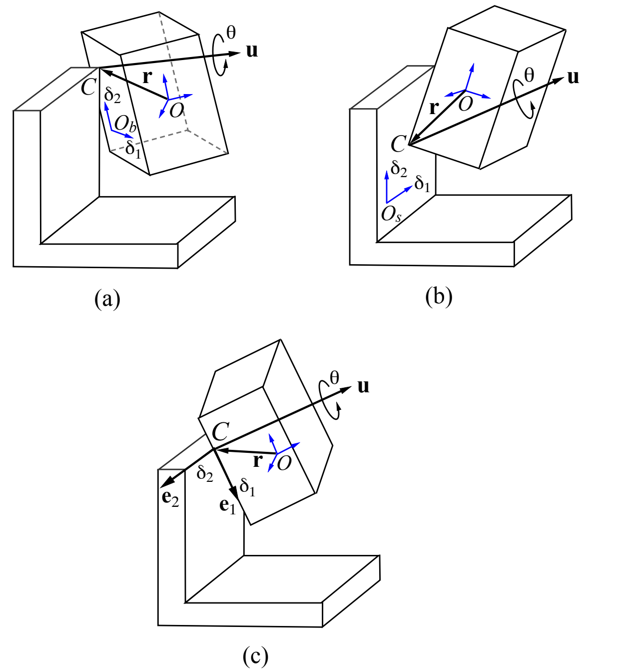


Fig. 1. Configuration variables for single-point principal contacts. (a) Face-vertex contact. (b) Vertex-face contact. (c) Edge-edge cross contact.

A. Orientational Variation

The relative orientation of the rigid body can be described by a 3×3 orthogonal matrix $\mathbf{R}$.

Consider a rotation about an arbitrary axis $\mathbf{u}$ with angle θ. The rotation matrix associated with this configuration change can be obtained by Rodrigues' formula [22]:

$$\mathbf{R}(\mathbf{u}, \theta) = \cos\theta \mathbf{I} + (1 - \cos\theta)\mathbf{u}\mathbf{u}^T + \sin\theta [\mathbf{u}\times] \qquad (5)$$

where $\mathbf{I}$ is the 3×3 identity matrix and $[\mathbf{u}\times]$ denotes the anti-symmetric matrix associated with the cross product operation. Since only small orientational variation is considered, the angular magnitude θ is small ($\leq 5°$). Thus the rotation matrix $\mathbf{R}$ in (5) can be accurately approximated by:

$$\mathbf{R}(\mathbf{u}, \theta) = \mathbf{I} + \sin\theta [\mathbf{u}\times]. \qquad (6)$$

Finite variation from an initial configuration can be described by placing bounds on the maximum angular magnitude $0 \leq \theta \leq \theta_M$ and with no bounds on the direction of the rotation axis, $\mathbf{u}$. Because $\mathbf{u}$ is arbitrary, for a centered coordinate frame with maximum angular variation $\Delta\theta$, the bound for the angular magnitude $\theta_M = \frac{1}{2}\Delta\theta$.

B. Translational Variation

For bodies in contact at a single point, the location of the contact point can be described by two parameters $\delta = (\delta_1, \delta_2)$. The meaning of these variables changes for the different principal contacts.

For face-vertex ($\{f\text{-}v\}$) contact, a 2-dimensional coordinate frame O_b is established on the held body in the plane of the contact face. Two orthogonal coordinates (δ_1, δ_2) are used to describe translational variation of the rigid body within this contact state as shown in Fig. 1a.

For vertex-face ($\{v\text{-}f\}$) contact, a 2-dimensional coordinate frame O_s is established on the stationary part in the plane of the contact face. Again, two orthogonal coordinates (δ_1, δ_2) are used to describe the translational variation of the rigid body within this contact state as shown in Fig. 1b.

For edge-edge cross ($\{e\text{-}e\}_c$) contact, as shown in Fig. 1c, two translational non-orthogonal coordinates (δ_1, δ_2) are chosen to describe translational variation along edges $\mathbf{e}_1$ and $\mathbf{e}_2$.

Since finite configuration variation is considered, for each contact state, the variation of each δ_i is bounded. By appropriately choosing the coordinate origin (at a central location of contact), the bounds for δ_i can be written as:

$$-\delta_{M_i} \leq \delta_i \leq \delta_{M_i}.$$

In summary, configuration variation for each single-point contact state is given by $\mathbf{q} = (\delta_1, \delta_2, \mathbf{u}, \theta)$.

III. Sufficient Condition for Face-Vertex Contact

As shown in Section II-A, the relative configuration of the bodies for face-vertex contact is described by the translation variables (δ_1, δ_2) and orientational variables $(\mathbf{u}, \theta)$. We prove that, if an admittance matrix $\mathbf{A}$ satisfies a set of conditions at the "boundary" points, then the $\mathbf{A}$ matrix ensures error-reducing motion for all intermediate configurations $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$ and $\theta \in [0, \theta_M]$ (regardless of the direction of rotation) for all values of friction in the range specified.

A. Error-Reduction Function

As shown in [17], for all face-vertex cases, the direction of the normal force is constant in the body frame and the normal wrench $\mathbf{w}_n$ is a function of only the translational variables (δ_1, δ_2). Thus, on the body, a coordinate frame can be selected such that the surface normal $\mathbf{n} = [0, 0, 1]^T$.

Since the direction of friction is perpendicular to $\mathbf{n}$ in the body frame, the unit vector $\mathbf{t}$ in the direction of the friction force can be expressed in the form:

$$\mathbf{t} = [\cos\psi,\ \sin\psi,\ 0]^T \qquad (7)$$

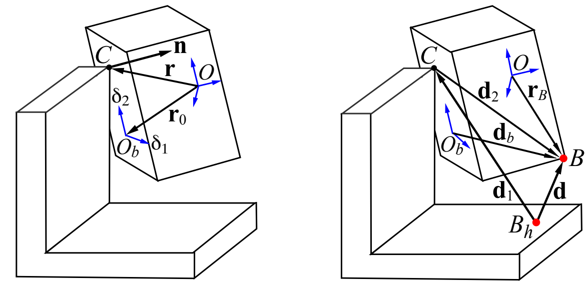
where ψ is a variable in $[0, 2\pi]$. From (4), it can be seen that the unit wrench associated with friction, $\mathbf{w}_t$, is a function of δ_i and ψ.

As shown in [17], the position vector to the contact point, $\mathbf{r}$ can be expressed as:

$$\mathbf{r} = \mathbf{r}_0 + \boldsymbol{\delta}' \qquad (8)$$

where $\mathbf{r}_0$ is the position vector from the body frame's origin O to the origin of the centrally located coordinate frame O_b, and $\boldsymbol{\delta}' = [\delta_1, \delta_2, 0]'$. The error-measure line vector $\mathbf{d}$ associated with point B can be expressed as:

$$\mathbf{d}(\delta, \theta) = \begin{bmatrix} \mathbf{R}\mathbf{d}_1' \\ \mathbf{r}_B \times \mathbf{R}\mathbf{d}_1' \end{bmatrix} + \begin{bmatrix} \mathbf{d}_b' \\ \mathbf{r}_B \times \mathbf{d}_b' \end{bmatrix} - \begin{bmatrix} \boldsymbol{\delta}' \\ \mathbf{r}_B \times \boldsymbol{\delta}' \end{bmatrix} \qquad (9)$$



(a) (b)

Fig. 2. Face-vertex contact. (a) Contact force in the body frame. (b) Error-measure vector $\mathbf{d}$ in the body frame.

where $\mathbf{d}_1'$, $\mathbf{d}_b'$ and $\mathbf{r}_B$ are constant vectors illustrated in Fig. 2b and $\mathbf{R}$ is the rotation matrix in (6).

Thus, for any intermediate configuration $(\delta_1, \delta_2, \theta)$, using (6), (7) and (9), the error-reduction function F_{1p} in (3) can be expressed as a function of $(\delta_1, \delta_2, \mathbf{u}, \theta)$ and ψ.

In the following, for an arbitrary wrench (6D line vector) $\mathbf{s}$, we denote $\mathbf{s}_\mathbf{u}$ as the cross product operation of $\mathbf{u}$ on $\mathbf{s}$, i.e., if $\mathbf{s}$ has the form:

$$\mathbf{s} = \begin{bmatrix} \mathbf{a} \\ \mathbf{r} \times \mathbf{a} \end{bmatrix}, \quad \text{then} \quad \mathbf{s}_\mathbf{u} = \begin{bmatrix} \mathbf{u} \times \mathbf{a} \\ \mathbf{r} \times (\mathbf{u} \times \mathbf{a}) \end{bmatrix}. \qquad (10)$$

Then, the error-reduction function can be written in the form:

$$\begin{aligned} F_{1p}(\delta, \theta) \ =\ & (\mathbf{d}_0 - \boldsymbol{\delta})^T (\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\, (\mathbf{w}_n + \mu \mathbf{w}_t) + \\ & \mathbf{d}_{1\mathbf{u}}^T (\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\, (\mathbf{w}_n + \mu \mathbf{w}_t) \sin\theta \qquad (11) \end{aligned}$$

where $\mathbf{d}_0$ is a constant 6-vector and the subscript $\mathbf{u}$ of a line vector indicates the cross product operation of $\mathbf{u}$ on the vector [as defined in (10)].

Now consider the matrix norm of the 6-vector $\mathbf{d}_{1\mathbf{u}}$. Since $\mathbf{u}$ is a unit vector, it can be shown $\|\mathbf{d}_{1\mathbf{u}}\| \leq \|\mathbf{d}_1\|$. Thus, the last term in (11)

$$\begin{aligned} & \mathbf{d}_{1\mathbf{u}}^T (\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\, (\mathbf{w}_n + \mu \mathbf{w}_t) \sin\theta \\ \leq\ & \|\mathbf{d}_1\|\, \|(\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\|\, (\|\mathbf{w}_n\| + \mu \|\mathbf{w}_t\|) \sin\theta_M \\ \leq\ & M \sin\theta_M \end{aligned}$$

where $M = \|\mathbf{d}_1\|\, \|(\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\|\, (\|\mathbf{w}_n\| + \mu_M \|\mathbf{w}_t\|)$ and the norms used are the conventional matrix norms. Note that in a specified coordinate frame, M is constant.

Now consider the first term in (11):

$$f = (\mathbf{d}_0 - \boldsymbol{\delta})^T (\mathbf{v}_0 \mathbf{w}_n^T - \mathbf{v}_0^T \mathbf{w}_n \mathbf{I})\, \mathbf{A}\, (\mathbf{w}_n + \mu \mathbf{w}_t).$$

Since $\mathbf{w}_n$ and $\mathbf{w}_t$ only contain linear terms in δ_i, f is a third order polynomial in δ_1 and δ_2. Note that unlike the frictionless case [17], the function f contains linear terms in $\cos\psi$ and $\sin\psi$ (in $\mathbf{w}_t$).

Similar to the method used for the frictionless case, we construct a new function:

$$F(\delta_1, \delta_2, \mu) = f + M \sin\theta_M \qquad (12)$$

then F is a third order polynomial in δ_1 and δ_2, and for all intermediate configurations and for $\mu \in [0, \mu_M]$,

$$F_{1p} \leq F(\delta_1, \delta_2, \mu).$$

B. Sufficient Conditions for Error-Reduction

In order to obtain sufficient conditions, we consider the "more positive" function defined in (12). The third order polynomial can be written in the form:

$$F(\delta_1,\delta_2) = f_1\delta_1^3 + f_2\delta_1^2\delta_2 + f_3\delta_1\delta_2^2 + f_4\delta_2^3 + f_5\delta_1^2 + f_6\delta_1\delta_2 + f_7\delta_2^2 + f_8\delta_1 + f_9\delta_2 + f_0 \quad (13)$$

where each f_i has the form:

$$f_i = g_{i1} + g_{i2}\cos\psi + g_{i3}\sin\psi. \quad (14)$$

Consider a single-variable function of δ_2 defined by:

$$f_{\delta_2} = F(0,\delta_2) = f_4\delta_2^3 + f_7\delta_2^2 + f_9\delta_2 + f_0.$$

Note that although the above equation has the same form as defined in [17] for frictionless case, by (14) the f_i's are functions of ψ. For any given μ, denote:

$$\begin{aligned} f_{Mi}(\mu) &= \|g_{i1}\| + \|g_{i2}\| + \|g_{i3}\|, \\ f_{M\delta_2}(\mu) &= \max\{|f_{M4}|, |f_{M7}|, |f_{M9}|\}, \\ f_m(\mu) &= \min_{|\delta_2|\le\delta_{M_2}} \{|f_{\delta_2}|\}, \\ c_M(\mu) &= \max_{|\delta_2|\le\delta_{M_2}} \{|f_{M1}|, |f_{M2}\delta_2 + f_{M5}|, \\ &\qquad |f_{3M}\delta_2^2 + f_{6M}\delta_2 + f_{8M}|\}, \end{aligned}$$

then, it can be proved that if

$$\frac{f_m(\mu)}{c_M(\mu) + f_m(\mu)} \ge \delta_{M_1}, \quad (15)$$

then, F_{1p} has no root for all $\delta_1 \in [-\delta_{M_1}, \delta_{M_1}]$ and $\delta_2 \in [-\delta_{M_2}, \delta_{M_2}]$. Therefore, for a given μ, if at one configuration the error reduction condition is satisfied, condition (15) guarantees error-reducing motion for all configurations.

Since each f_i in (13) is a linear function of μ, the inequality (15) at the two boundary values of friction 0 and μ_M ensures that F_{1p} has no root for all configurations considered and for all $\mu \in [0, \mu_M]$. Thus we have:

Proposition 1. For a face-vertex contact state, if: i) at the configuration $(\delta_1, \delta_2, \theta) = (0, 0, 0)$, the admittance satisfies the error reduction condition (2), and ii) condition (15) is satisfied for the polynomial (13) at the two boundary values of friction coefficients $\mu = 0$ and $\mu = \mu_M$, then the admittance will satisfy the error reduction conditions for all $\mu \in [0, \mu_M]$ and all configurations bounded by $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$ and $\theta \in [0, \theta_M]$ in any direction. □

IV. SUFFICIENT CONDITION FOR VERTEX-FACE CONTACT

For this type of contact state, similar to the frictionless case, the configuration variations in orientation and translation can be considered separately. Here, we modify the procedure used for frictionless contact [17] to include the influence of friction.

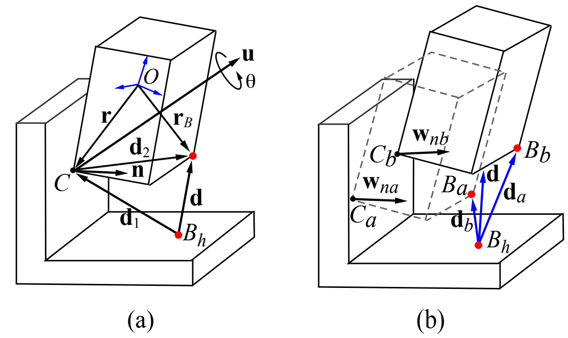


Fig. 3. Vertex-face contact state. (a) Orientational variation. (b) Translational variation.

A. Configuration Variation in Orientation

Consider a rotation given by an angle change $\theta \in [0, \theta_M]$ about an axis $\mathbf{u}$. If we denote $\mathbf{n}_0$ as the surface normal associated with $\theta = 0$, then in the body coordination frame, the surface normal associated with varying $(\mathbf{u}, \theta)$ is:

$$\mathbf{n}_\theta = \mathbf{R}(\theta)\mathbf{n}_0 \quad (16)$$

where $\mathbf{R}$ is the rotation matrix having the form of (5).

Thus, the unit wrench associated with contact is:

$$\mathbf{w} = \mathbf{w}_n(\theta) + \mu\mathbf{w}_t = \begin{bmatrix} \mathbf{R}\mathbf{n}_0 \\ \mathbf{r}\times\mathbf{R}\mathbf{n}_0 \end{bmatrix} + \mu \begin{bmatrix} \mathbf{R}\mathbf{t}_0 \\ \mathbf{r}\times\mathbf{R}\mathbf{t}_0 \end{bmatrix} \quad (17)$$

where $\mathbf{r}$ is the position vector from the origin of the body frame to the contact point (constant in body frame) and $\mathbf{t}_0$ is an arbitrary unit vector in the original tangent plane.

Using the same description presented in [17], the error-measure line-vector $\mathbf{d}$ is:

$$\mathbf{d}(\theta) = \begin{bmatrix} \mathbf{d}' \\ \mathbf{r}_B \times \mathbf{d}' \end{bmatrix} = \begin{bmatrix} \mathbf{R}\mathbf{d}'_1 \\ \mathbf{r}_B \times \mathbf{R}\mathbf{d}'_1 \end{bmatrix} + \begin{bmatrix} \mathbf{d}'_2 \\ \mathbf{r}_B \times \mathbf{d}'_2 \end{bmatrix} \quad (18)$$

where $\mathbf{r}_B$ is the position vector from the body frame origin to point B and where $\mathbf{d}'_1$ is the position 3-vector from B_h to the contact point C and $\mathbf{d}'_2$ is the position 3-vector from C to point B (Fig. 3a).

Substituting (17) and (18) into (3) and using (6), the error-reduction function can be expressed as a function of $(\mathbf{u}, \theta)$ in the form:

$$F_{1p}(\theta) = F_{1p}(0) + F_1\sin\theta + F_2\sin^2\theta + F_3\sin^3\theta \quad (19)$$

where

$$\begin{aligned} F_1 &= \mathbf{d}_{1\mathbf{u}}^T(\mathbf{v}_0\mathbf{w}_{n0}^T - \mathbf{v}_0^T\mathbf{w}_{n0}\mathbf{I})\mathbf{A}\mathbf{w}_0 + \mathbf{d}_0^T(\mathbf{v}_0\mathbf{w}_{n0}^T - \\ &\quad \mathbf{v}_0^T\mathbf{w}_{n0}\mathbf{I})\mathbf{A}\mathbf{w}_{0\mathbf{u}} + \mathbf{d}_0^T(\mathbf{v}_0\mathbf{w}_{n0\mathbf{u}}^T - \mathbf{v}_0^T\mathbf{w}_{n0\mathbf{u}}\mathbf{I})\mathbf{A}\mathbf{w}_0, \\ F_2 &= \mathbf{d}_{1\mathbf{u}}^T(\mathbf{v}_0\mathbf{w}_{n0}^T - \mathbf{v}_0^T\mathbf{w}_{n0}\mathbf{I})\mathbf{A}\mathbf{w}_{0\mathbf{u}} + \mathbf{d}_0^T(\mathbf{v}_0\mathbf{w}_{n0\mathbf{u}}^T - \\ &\quad \mathbf{v}_0^T\mathbf{w}_{n0\mathbf{u}}\mathbf{I})\mathbf{A}\mathbf{w}_{0\mathbf{u}} + \mathbf{d}_{1\mathbf{u}}^T(\mathbf{v}_0\mathbf{w}_{n0\mathbf{u}}^T - \mathbf{v}_0^T\mathbf{w}_{n0\mathbf{u}}\mathbf{I})\mathbf{A}\mathbf{w}_0, \\ F_3 &= \mathbf{d}_{1\mathbf{u}}^T(\mathbf{v}_0\mathbf{w}_{n0\mathbf{u}}^T - \mathbf{v}_0^T\mathbf{w}_{n0\mathbf{u}}\mathbf{I})\mathbf{A}\mathbf{w}_{0\mathbf{u}} \end{aligned}$$

where $\mathbf{w}_0 = \mathbf{w}_{n0} + \mu\mathbf{w}_{t0}$ with $\mathbf{w}_{n0}$ being the initial normal wrench and $\mathbf{w}_{t0}$ being the unit wrench in the initial tangent plane, $\mathbf{d}_0$ is the error measure line vector when $\theta = 0$ and where the subscript $\mathbf{u}$ of a line vector indicates the cross product operation of $\mathbf{u}$ on the vector as defined in (10).

Note that although $\mathbf{w}_t$ is unknown, its norm is bounded by a number M_t.

To achieve error reduction at all other orientations considered, $F_{1p}(\theta)$ must be negative for $\theta \in [0, \theta_M]$ and an arbitrary rotation axis $\mathbf{u}$ and arbitrary friction direction $\mathbf{t}$. Since $\mathbf{u}$ is a unit vector, the bounds for F_i in (19) can be obtained.

If we denote:

$$M = \|\mathbf{d}_0\| \cdot \|(\mathbf{v}_0\mathbf{w}_{n0}^T - \mathbf{v}_0^T\mathbf{w}_{n0}\mathbf{I})\mathbf{A}\| \cdot (\|\mathbf{w}_{n0}\| + \mu_M M_t) \quad (20)$$

where the norm used is the conventional matrix norm, then,

$$|F_1| \leq 3M, \quad |F_2| \leq 3M, \quad |F_3| \leq M.$$

Consider the new function constructed by:

$$F = F_{1p}(0) + 3M\sin\theta_M + 3M\sin^2\theta_M + M\sin^3\theta_M.$$

Then, for $\theta \in [0, \theta_M]$ with an arbitrary rotation axis, we have

$$F_{1p}(\mathbf{u}, \theta) \leq F. \quad (21)$$

Thus, if

$$F = F_{1p}(0) + 3M\sin\theta_M + 3M\sin^2\theta_M + M\sin^3\theta_M < 0, \quad (22)$$

then $F_{1p}(\mathbf{u}, \theta) < 0$ for all orientational variations considered.

B. Configuration Variation in Translation

When only translational variation of configuration is considered, the contact force and motion response do not change in the body frame.

For a given orientation, the configuration of the body can be determined by the location (δ_1, δ_2) of the vertex C. Thus, if at two configurations $(-\delta_{M_1}, \delta_2)$ and (δ_{M_1}, δ_2) the error reduction condition is satisfied, then the error reduction condition must be satisfied for all intermediate configurations (δ_1, δ_2) with $\delta_1 \in [-\delta_{M_1}, \delta_{M_1}]$. The same result holds true for variation in δ_2 while δ_1 is constant.

C. General Case

By the same reasoning presented in [17], the results presented in IV-A and IV-B can be generalized to all intermediate vertex-face contact configurations involving both translational and orientational variations from configurations at which the conditions were imposed.

Note that since the bound M in (20) is defined at $\mu = \mu_M$, the maximum value of the coefficient of friction, inequality (21) holds true for all $\mu \in [0, \mu_M]$. Thus, we have:

Proposition 2. For a vertex-face contact state with variation of orientation $[0, \theta_M]$ and variation of translation $[-\delta_{M_i}, \delta_{M_i}]$, if inequality (22) is satisfied at the four translational boundary points $(\pm\delta_{M_1}, \pm\delta_{M_2})$, then the admittance will satisfy the error reduction condition for all $\mu \in [0, \mu_M]$ and all configurations bounded by $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$, and $\theta \in [0, \theta_M]$ in any rotation direction. □

Thus, for a vertex-face contact state, to ensure that the motion response due to contact is error reducing for all configurations considered, only four conditions need to be satisfied.

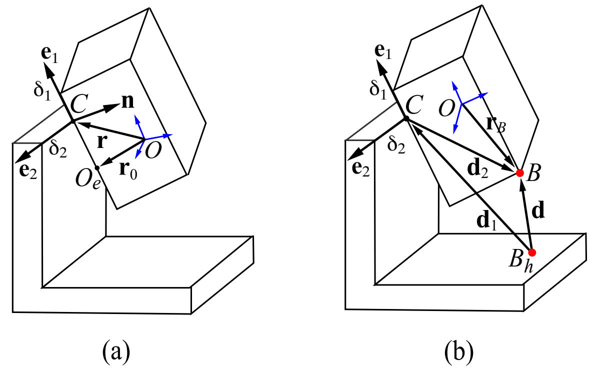


Fig. 4. Edge-edge cross contact. (a) Contact force in the body frame. (b) Error-measure vector $\mathbf{d}$ in the body frame.

V. Sufficient Conditions for Edge-Edge Cross Contact

Below, for "edge-edge cross" contact, we identify the set of conditions that, when satisfied for a given admittance matrix $\mathbf{A}$ at the "boundary" points, ensures error-reducing motion for all intermediate configurations $\theta \in [0, \theta_M]$, $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$, and $\mu \in [0, \mu_M]$.

A. Error-Reduction Function

For all edge-edge cross contact cases, the direction of the normal force depends only on the orientational variation while the position vector of the contact point, $\mathbf{r}$, depends only on the translational variation along the contact edge of the held body $\mathbf{e}_1$ (Fig. 4a). For arbitrary (δ_1, δ_2), $\mathbf{r}$ can be expressed as:

$$\mathbf{r} = \mathbf{r}_0 + \delta_1\mathbf{e}_1$$

where $\mathbf{r}_0$ is a vector from the body frame to a centrally located point O_e on the edge $\mathbf{e}_1$ (constant).

Let $\mathbf{n}_0$ be the normal direction of the contact force determined by $\mathbf{e}_1 \times \mathbf{e}_2$ at the initial position. Then, for rotation $\mathbf{R}$, the normal direction is determined by $\mathbf{e}_1 \times \mathbf{R}\mathbf{e}_2$. If the rotation angle θ is small, the rotation angle of $\mathbf{n}$, β is also small, i.e., $|\beta| \leq \theta_M$. Suppose the rotation axis and angle of $\mathbf{n}$ associated with rotation of $\mathbf{e}_2$ are $\mathbf{s}$ and β, then by (6)

$$\mathbf{n} = \mathbf{n}_0 + \mathbf{s} \times \mathbf{n}_0 \sin\beta.$$

Thus, $\mathbf{w}_n$ and $\mathbf{d}$ (in Fig. 4b) can be expressed in terms of ($\mathbf{s}$, $\sin\beta$) and ($\mathbf{u}$, $\sin\theta$) respectively:

$$\begin{aligned} \mathbf{w}_n &= \mathbf{w}_0 - \mathbf{w}_{0\mathbf{s}}\sin\beta \\ \mathbf{d} &= \mathbf{d}_1 + \mathbf{d}_2 + \boldsymbol{\delta}_1 + \boldsymbol{\delta}_2 + (\mathbf{d}_1 + \boldsymbol{\delta}_2)_{\mathbf{u}}\sin\theta \end{aligned}$$

where $\mathbf{w}_0$ is the wrench when $\theta = 0$ and $\boldsymbol{\delta}_i$ is defined as:

$$\boldsymbol{\delta}_i = \delta_i \begin{bmatrix} \mathbf{e}_i \\ \mathbf{r}_B \times \mathbf{e}_i \end{bmatrix}, \quad i = 1, 2.$$

For friction wrench $\mathbf{w}_t$, we consider all possible directions. Thus no constraint is imposed on $\mathbf{w}_t$.

Substituting the above $\mathbf{w}_n$ and $\mathbf{d}$ into (3) and sorting the coefficients of $\sin^i\beta$ and $\sin^{i-1}\beta\sin\theta$, the error-reduction function can be expressed as:

$$F_{1p}(\delta,\theta) = F_0 + F_1(\sin\theta + a_1\sin\beta) + F_2(\sin\theta\sin\beta + a_2\sin^2\beta) + F_3(\sin\theta\sin^2\beta + a_3\sin^3\beta) \quad (23)$$

where:

$$\begin{aligned} F_0 &= (\mathbf{d}_1+\mathbf{d}_2+\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)^T(\mathbf{v_0}\mathbf{w}_0^T - \mathbf{v}_0^T\mathbf{w}_0\mathbf{I})\mathbf{A}\mathbf{w}, \\ F_1 &= -(\mathbf{d}_1+\mathbf{d}_2+\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)^T(\mathbf{v_0}\mathbf{w}_{0\mathbf{s}}^T - \mathbf{v}_0^T\mathbf{w}_{0\mathbf{s}}\mathbf{I})\mathbf{A}\mathbf{w} \\ &\quad -(\mathbf{d}_1+\mathbf{d}_2+\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)^T(\mathbf{v_0}\mathbf{w}_0^T - \mathbf{v}_0^T\mathbf{w}_0\mathbf{I})\mathbf{A}(\mathbf{w}_{0\mathbf{s}}+\mu\mathbf{w}_t) \\ &\quad +(\mathbf{d}_1+\boldsymbol{\delta}_2)_{\mathbf{u}}^T(\mathbf{v_0}\mathbf{w}_0^T - \mathbf{v}_0^T\mathbf{w}_0\mathbf{I})\mathbf{A}\mathbf{w}, \\ F_2 &= (\mathbf{d}_1+\mathbf{d}_2+\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)^T(\mathbf{v_0}\mathbf{w}_{0\mathbf{s}}^T - \mathbf{v}_0^T\mathbf{w}_{0\mathbf{s}}\mathbf{I})\mathbf{A}(\mathbf{w}_{0\mathbf{s}}+\mu\mathbf{w}_t) \\ &\quad -(\mathbf{d}_1+\boldsymbol{\delta}_2)_{\mathbf{u}}^T(\mathbf{v_0}\mathbf{w}_{0\mathbf{s}}^T - \mathbf{v}_0^T\mathbf{w}_{0\mathbf{s}}\mathbf{I})\mathbf{A}\mathbf{w} \\ &\quad -(\mathbf{d}_1+\boldsymbol{\delta}_2)_{\mathbf{u}}^T(\mathbf{v_0}\mathbf{w}_0^T + \mathbf{v}_0^T\mathbf{w}_0\mathbf{I})\mathbf{A}(\mathbf{w}_{0\mathbf{s}}+\mu\mathbf{w}_t), \\ F_3 &= (\mathbf{d}_1+\boldsymbol{\delta}_2)_{\mathbf{u}}^T(\mathbf{v_0}\mathbf{w}_{0\mathbf{s}}^T - \mathbf{v}_0^T\mathbf{w}_{0\mathbf{s}}\mathbf{I})\mathbf{A}(\mathbf{w}_{0\mathbf{s}}+\mu\mathbf{w}_t). \end{aligned}$$

Similar to the results presented in V-A.2, because $\mathbf{u}$ and $\mathbf{t}$ are unit vectors, each F_i in the above equations is bounded. Let F_{M_i} be a bound for $|F_i|$ evaluated at $\mu = \mu_M$, i.e,

$$\max\{\ |F_i|_{\mu=\mu_M}\ \} \le F_{Mi} \quad i = 1,2,3. \quad (24)$$

Since $|\beta| \le \theta_M$, the terms associated with F_i in (23) are bounded:

$$|\sin\theta + a_i\sin\beta\sin^{i-1}\theta| \le (1+|a_i|)\sin^i\theta_M, \quad i = 1,2,3.$$

Now define a new function:

$$G = F_0 + G_{M_1}\sin\theta_M + G_{M_2}\sin^2\theta_M + G_{M_3}\sin^3\theta_M \quad (25)$$

where

$$G_{M_1} = F_{M_i}(1+|a_i|)\sin^i\theta_M, \quad i = 1,2,3.$$

Then, G is a linear function in δ_1 and δ_2 and for all $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$ and $\theta \in [0, \theta_M]$,

$$F_{1p} \le G. \quad (26)$$

Thus, if G is negative for $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$, then F_{1p} must be negative for all $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$ and for all rotations with $\theta \le \theta_M$ in any direction. Since G is a linear function in δ_1 and δ_2, $G < 0$ for all δ_i's in the bounded area if and only if, at the four boundary points $(\pm\delta_{M_1}, \pm\delta_{M_2})$, $G < 0$.

Note that because the bounds F_{Mi} defined in (24) are obtained at the maximum value of coefficient of friction (μ_M), (26) holds true for all $\mu \in [0, \mu_M]$. Thus, we have:

Proposition 3. For an edge-edge cross contact state with variation of orientation $[0, \theta_M]$ and variation of translation $[-\delta_{M_i}, \delta_{M_i}]$, if at the four translational boundary points $(\pm\delta_{M_1}, \pm\delta_{M_2})$ the function G defined in (25) is negative, then the admittance will satisfy the error reduction condition for all $\mu \in [0, \mu_M]$ and for all configurations bounded by $\delta_i \in [-\delta_{M_i}, \delta_{M_i}]$ and for all misalignments about an arbitrary direction with angle $\theta \le \theta_M$. □

VI. DISCUSSION AND SUMMARY

In this paper, error-reduction of a single point on the held body is considered when evaluating error-reduction of the held body. For spatial motion, the error-measure at a single point on the held part is not adequate. The set of error-reduction conditions satisfied at a single point does not ensure an error-reducing motion of the whole body. In practice, multiple points should be selected as reference points for error-measures. If, for example, n points on the body periphery are judiciously selected as reference points, then the error-reduction conditions must be satisfied for each of the n error measures. Therefore, the associated conditions (Propositions 1-3) must be applied to all of the n points. Note that increasing the number of reference points makes the error-reduction conditions more restrictive. For spatial assembly, error-reducing motion at three non-collinear reference points on the body periphery would be necessary and sufficient to ensure that the parts are properly mated.

For each PC considered, the conditions ensure error-reducing motion only within the same PC. In order to achieve reliable assembly in tasks that involve multiple PCs, conditions for each of the PC's that may occur in the assembly must be imposed on the admittance simultaneously.

The main contribution of this paper is the development of a method used to account for frictional contact without solving the complicated nonlinear equations of [19]. Here we assume that the error-reduction conditions are satisfied for all possible directions of friction in the tangent plane. Then by evaluating the bounds of simplified functions, sufficient conditions are obtained. When evaluating the bounds of these functions, the maximum/minimum values are considered. Thus the obtained conditions ensure error-reducing motion for all coefficients of friction in the range specified. Since the conditions consider friction in an arbitrary direction in the contact tangent plane, the conditions obtained are conservative. Also, since the methods presented are for general polyhedral parts, the bounds of functions are conservatively estimated. For a real problem, since the geometry of the parts is given, the functions in (13), (19) and (21) can be expressed more explicitly. The more accurate bound values for these functions could be calculated. Thus, less conservative conditions could be obtained.

In summary, we have presented a set of conditions for admittance selection for force-guided assembly of any two polyhedral rigid bodies despite bounded friction. We have shown that, for single-PC contact states, the admittance control law can be selected based on imposed behavior at a *finite* number of configurations. If the error reduction conditions are satisfied at these configurations, the error reduction conditions will be satisfied for all intermediate configurations for all values of friction less than the specified upper bound.

In future work, more general admittance selection problems involving contact at multiple locations with friction will be investigated.

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