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Variable Stiffness Springs for Energy Storage Applications

Sung Y. Kim, Tiange Zhang, and David J. Braun

***Abstract*— Theory suggests an inverse relation between the stiffness and the energy storage capacity for linear helical springs: reducing the active length of the spring by 50% increases its stiffness by 100%, but reduces its energy storage capacity by 50%. State-of-the-art variable stiffness actuators used to drive robots are characterized by a similar inverse relation, implying reduced energy storage capacity for increased spring stiffness. This relation limits the potential of the variable stiffness actuation technology when it comes to human performance augmentation in natural tasks, e.g., jumping, weight-bearing and running, which may necessitate a spring exoskeleton with large stiffness range and high energy storage capacity. In this paper, we theoretically show that the trade-off between stiffness range and energy storage capacity is not fundamental; it is possible to develop variable stiffness springs with simultaneously increasing stiffness and energy storage capacity. Consistent with the theory, we experimentally show that a controllable volume air spring, has a direct relation between its stiffness range and energy storage capacity. The mathematical conditions presented in this paper may be used to develop actuators that could bypass the limited energy storage capacity of current variable stiffness spring technology.**

I. INTRODUCTION

Variable stiffness actuators [1] used in robotics [2], prostheses [3], and human augmentation [4], provide stiffness modulation for stable robot-environment interaction [5] and safe human-robot interfacing [6]. Variable stiffness spring actuators [7], [8] may also be used to accumulate elastic energy while increasing stiffness under limited spring deformation. This method of energy storage could be useful for human performance augmentation [9]. For example, given a high enough spring stiffness, a large amount of gravitational potential energy may be stored within a limited spring deformation, which could be subsequently useful to jump higher, walk with a heavy backpack, or run faster beyond what is possible using a more conventional series elastic actuator. However, energy storage within a limited deformation range of an actuator requires high stiffness, which reduces the energy storage capacity of most variable stiffness actuators.

Stiffness modulation may be achieved by extending the active length of the spring in antagonistic and spring-pretension based variable stiffness actuators [10], [11], reducing the active length of a helical spring [12] and leaf spring based actuators [8], [13], or changing the transmission ratio in variable moment arm actuators [14], [15]. In all aforementioned works, there is an inverse relation between the stiffness and the energy storage capacity of the actuator. Such an inverse relation entails that increasing stiffness reduces the energy storage capacity of the actuator. A notable exception is a previously designed variable moment arm actuator [16], where changing the mechanical advantage is used to increase stiffness, without reducing the energy storage capacity of the actuator. However, modifying the mechanical advantage of a kinematic mechanism attached to the spring cannot increase the energy storage capacity of the spring; increasing the energy storage capacity of a spring can only be done by changing the geometric or material property of the spring.

The range of stiffness and the energy storage capacity of a spring may be simultaneously increased by increasing the mass of the spring. For example, in coil springs and leaf springs, adding mass could be done by increasing the thickness of the spring. However, increasing the thickness of a coil or leaf spring is nontrivial in a practical design. While increasing stiffness may be theoretically done with zero net mechanical work [7], [8], [17], [18], the fundamental question that remains is how to simultaneously increase both the stiffness and the energy storage capacity of a variable stiffness spring that, at least theoretically, does not require mechanical work. Developing such an actuator remains elusive in both theoretical studies and practical application [19].

In this paper, we present a mathematical characterization of variable stiffness spring actuators that can simultaneously increase stiffness and energy storage capacity with negligible (theoretically zero) energy cost. We derive a set of analytical conditions that define the class of these actuators. Using these conditions, we identify a variable volume air spring as one of the simplest actuators that satisfies our analytical conditions. Using a variable volume air spring, we experimentally demonstrate simultaneous increase of stiffness and energy capacity by increasing the volume of the air spring at atmospheric pressure. In practice, increasing the volume of the air spring at atmospheric pressure requires negligible amount of mechanical work compared to the work done to deform the spring.

High energy capacity variable stiffness springs may be used to develop actuators for robots, that could reduce the weight and size of more conventional actuators and batteries [20], [21]. Variable stiffness springs with high energy capacity may also be useful for human performance augmentation, where, beyond providing stiffness modulation, they could store large amounts of energy to enable augmented humans bypass the limitations of their biological limb [9], [22].

Sung Y. Kim, Tiange Zhang, and David J. Braun are with the Department of Mechanical Engineering, Vanderbilt University, Nashville, Tennessee 37235, USA.
Email: sung.kim@vanderbilt.edu
Email: tiange.zhang@vanderbilt.edu
Email: david.braun@vanderbilt.edu

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II. Variable Stiffness Springs

Variable stiffness actuators use two motors; one to control the deformation, and another to set the stiffness of the spring [1], [2], [17]. In this work, we consider variable stiffness spring actuators [18], sometimes referred to as stiffness modulators [7], [8], which only provide stiffness modulation without changing the equilibrium position of the actuator. The conceptual model of a variable stiffness spring actuator is shown in Fig. 1.

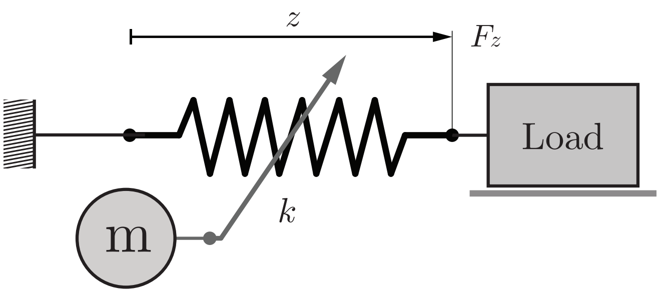

Fig. 1. Model of a variable stiffness spring. In this model, z denotes the length of the spring, F_z denotes the force by the spring, while k denotes the stiffness of the spring changed by the "motor" m.

A. Variable stiffness helical springs

The potential energy function of a helical extension spring, with changeable active length and active length dependent stiffness, is given by:

$$V(z,z_0) = \frac{1}{2}k(z_0)(z-z_0)^2, \tag{1}$$

where $z-z_0$ defines the deformation of the spring, while $k(z_0)$ is the stiffness of the spring. The latter relation may be justified by noting that the stiffness of a helical spring is the function of the number of active coils which, in turn, defines the active length z_0 of the spring [12].

B. Variable stiffness springs

The mathematical model of a variable stiffness spring which has changeable active length, and active length dependent stiffness, may be defined by its potential energy function:

$$V(z,z_0) = \sum_{m=0}^{+\infty}\sum_{n=-\infty}^{+\infty} C_{m,n}(z-z_0)^m z_0^n, \tag{2}$$

where the exponents $(m,n) \in \mathbb{N}\times\mathbb{Z}$ define the kinematic design of the spring, while $C_{m,n}$ encapsulates the geometric and physical parameters. This Laurent series representation of the potential energy function is similar to the one proposed in [23].

III. Design Conditions for the Ideal Variable Stiffness Spring

In this section, we introduce four analytical design conditions to define the model of a variable stiffness spring characterized by increased energy storage capacity with increased stiffness. We will use these conditions to find the exponents $(m,n) \in \mathbb{N}\times\mathbb{Z}$ in (2) that could be used to model an ideal variable stiffness spring designed for energy storage.

The first condition describes the behavior of a typical spring; it asserts zero force at zero deformation (Section III-A). The second and third conditions assert simultaneously increasing stiffness range and energy storage capacity (Sections III-B and III-C). Finally, the fourth condition ensures that no force is required to change the stiffness of the spring when the spring does not store energy (Section III-D). The last condition enables the stiffness and the energy storage capacity of the spring to be increased with theoretically zero mechanical work when the spring does not store energy.

A. Zero output force with zero deformation

When the spring is not deformed, i.e. when $z = z_0$, the force provided by the spring should be zero regardless of the stiffness:

$$\forall z_0 : F_z(z_0,z_0) = -\frac{\partial V}{\partial z}(z_0,z_0) = 0. \tag{3}$$

Condition (3) yields the following set of basis functions:

$$\mathscr{B}_1 = \{(z-z_0)^m z_0^n\}, \text{ where } (m,n) \in \{\mathbb{N}\times\mathbb{Z} : m \neq 1\}. \tag{4}$$

This condition indicates that the exponent of the deformation $(z-z_0)^m$ must be either zero or strictly greater than one. This condition is consistent with the potential energy function of standard helical springs, torsional springs, and leaf springs, as well as, the potential energy function of spring pretension based and variable moment arm based variable stiffness actuators [17]. We note that (4) does not impose a symmetric force-deformation relation, because such relation does not constitute a necessary condition for the practical realization of a spring. For example, helical extension springs do not have symmetric force-deformation relations, as they cannot be compressed. The following two conditions define springs for which the stiffness range and the energy storage capacity simultaneously increase, as the geometry of the spring changes.

B. Controllable increase in stiffness range

The following condition ensures that the range of achievable stiffness values of the spring increases as the geometry of the spring changes (as z_0 is increased):

$$\frac{\partial\,|k_{\max}-k_{\min}|}{\partial z_0} > 0, \tag{5}$$

where $k_{\max}$ and $k_{\min}$ are defined by:

$$k_{\max} = \max_{z\in[z_{\min},z_{\max}]} \frac{\partial^2 V}{\partial z^2}(z,z_0),\;\; k_{\min} = \min_{z\in[z_{\min},z_{\max}]} \frac{\partial^2 V}{\partial z^2}(z,z_0).$$

In order to present a reasonably simple representation of (5), we assume that the minimum stiffness is attained at zero deflection $z-z_0 = 0$ while the maximum stiffness is attained at the largest deflection $z = z_k \in \{z_{\min}, z_{\max}\}$. Under this assumption, the aforementioned inequality relation (5) leads to the following set of basis functions:

$$\mathscr{B}_2 = \{(z-z_0)^m z_0^n\}, \text{ where } (m,n) \in \{\mathbb{N}\times\mathbb{Z} : m \geq 2, \exists m > 2 \\ \text{sgn}[C_{m,n}]\text{sgn}[(z_k-z_0)^{m-3}][(z_k-z_0)n-(m-2)z_0] > 0\} \tag{6}$$

This condition is sufficient but not necessary to achieve (5).

C. Controllable increase in energy storage capacity

We aim to design a spring that has increased energy storage capacity as its stiffness increases. Since the previous condition ensures that the stiffness increases with z_0, the spring should store more elastic potential energy as z_0 increases. This assertion leads to the following condition:

$$\frac{\partial V_{\max}}{\partial z_0} = \frac{\partial}{\partial z_0} \max_{z\in[z_{\min},z_{\max}]} V(z,z_0) > 0. \tag{7}$$

Assuming that the maximum potential energy is attained at $z = z_V \in \{z_{\min}, z_{\max}\}$, the inequality can be reduced to the following set of basis functions:

$$\mathscr{B}_3 = \{(z-z_0)^m z_0^n\}, \text{ where } (m,n) \in \{\mathbb{N}\times\mathbb{Z} : \\ \text{sgn}[C_{m,n}]\text{sgn}[z_V - z_0]^{m-1}\left(m+n-\frac{z_V}{z_0}n\right) < 0\}. \tag{8}$$

Similar to the condition derived in the previous section, (8) is sufficient but not necessary to achieve (7).

D. Zero input force with zero deformation

For variable stiffness springs that do not require energy to hold stiffness, the stiffness modulating actuator, e.g., the motor in Fig. 1, should experience zero force when the spring is not deformed, i.e., when it does not store energy [17]. This imposes the following condition:

$$\forall z_0 : F_{z_0}(z_0,z_0) = -\frac{\partial V}{\partial z_0}(z_0,z_0) = 0. \tag{9}$$

This equality relation implies the following set of basis functions:

$$\mathscr{B}_4 = \{(z-z_0)^m z_0^n\}, \text{ where } (m,n) \in \{\mathbb{N}\times\mathbb{Z} : m \geq 2\} \cup (0,0). \tag{10}$$

According to (10), the exponent of the deformation should be greater than one or both exponents must be zero.

IV. Design of a Spring using Analytical Conditions

Potential energy functions that describe the class of variable stiffness springs, ideal for energy storage applications, can be obtained by a linear combination of basis functions that satisfy the following conditions (3), (5), (7), and (9). The basis functions that satisfy these conditions are given by:

$$\mathscr{B}_{\text{VSS}} = \mathscr{B}_1 \cap \mathscr{B}_2 \cap \mathscr{B}_3 \cap \mathscr{B}_4. \tag{11}$$

An example of a simplest potential energy function – the one with the least number of basis functions and the lowest order of basis functions that belong to $\mathscr{B}_{\text{VSS}}$ is given by:

$$V(z,z_0) = C_{3,-2}\frac{(z-z_0)^3}{z_0^2}, \;\; \frac{1}{2}z_0 \leq z \leq z_0, \;\; C_{3,-2} < 0. \tag{12}$$

In the next section, we will present one practical design where the potential energy function is constructed using a linear combination of similar basis functions given by:

$$\mathscr{B}^*_{\text{VSS}} = \left\{\frac{(z-z_0)^2}{z_0}, \frac{(z-z_0)^3}{z_0^2}, \frac{(z-z_0)^4}{z_0^3}, \frac{(z-z_0)^5}{z_0^4}, \ldots\right\}. \tag{13}$$

V. Variable Volume Air Spring

An air spring with controllable volume, i.e., controllable air mass, is a device that can simultaneously increase stiffness and the maximum potential energy stored by the spring. Increasing the air mass can be done in two different ways. First, the air flow control valve can be opened to let air into the chamber by increasing the height of the piston when the chamber is at atmospheric pressure. Second, an air pump can be used to add air to the chamber. The important difference between these two methods is that the former can theoretically be done with zero net mechanical work, while the latter requires mechanical work in proportion to the air mass pumped into the cylinder and the pressure in the cylinder. Our work will focus on the former, since that method may be practically implemented using a small motor and a small battery because opening and closing the valve and moving the piston at atmospheric pressure requires limited amount of external energy.

A. Model of the Air Spring

Given the assumption that the air spring is compressed at a sufficiently low frequency, the compression of the air spring may be considered an isothermal process. Consequently, the potential energy function of the variable volume air spring is given by:

$$V(z,z_0) = F_{\text{atm}}\Big(z - z_0 - z_0\ln\left(\frac{z}{z_0}\right)\Big) = \sum_{p=2}^{\infty}(-1)^p\frac{F_{\text{atm}}}{p!}\frac{(z-z_0)^p}{z_0^{p-1}} \tag{14}$$

where F_{atm} is the atmospheric pressure, while the length of the cylinder and the stroke length are given by:

$$z_0 \in [z_{\min}, z_{\max}] \text{ and } z \in [z_{\min}, z_0]. \tag{15}$$

One can confirm that (14) satisfies the analytical design conditions given in (11).

Based on (14), the force-deformation characteristic of the spring is defined by:

$$F(z,z_0) = -\frac{\partial V(z,z_0)}{\partial z} = F_{\text{atm}}\Big(\frac{z_0}{z} - 1\Big), \;\; \forall z \in [z_{\min}, z_0] \tag{16}$$

where the initial atmospheric pressure inside the uncompressed air cylinder ensures $F(z_0,z_0) = 0$.

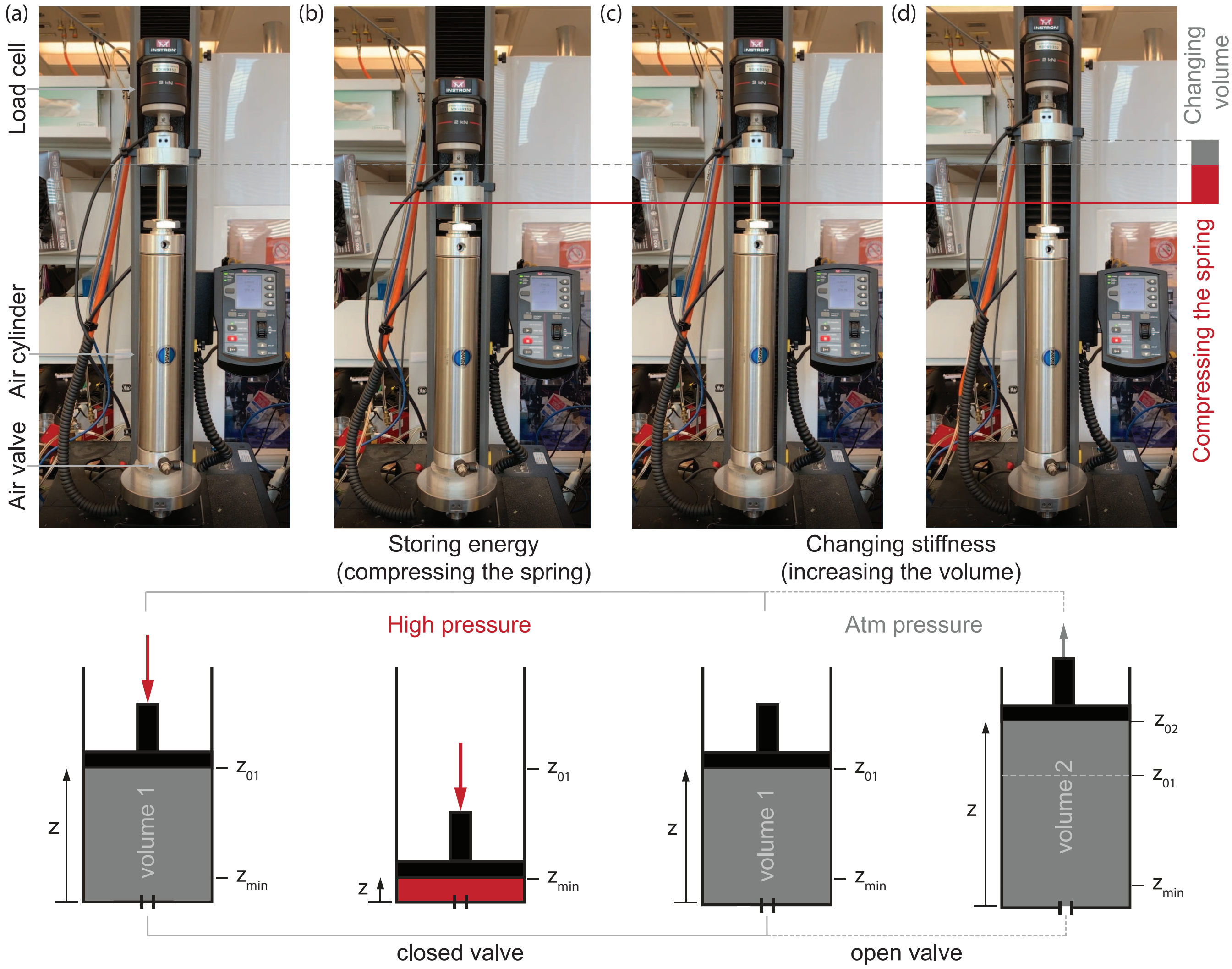

Fig. 2. Experimental set up of the controllable-volume air spring within one compression-expansion cycle. (*a-b*) Atmospheric air is being compressed from equilibrium height z_{01} to the minimum height z_{min} while the air flow control valve is closed. (*c*) The piston is then returned to the equilibrium height z_{01} (Volume 1) with the valve closed. (*d*) Once the piston reaches the equilibrium position, it is raised up to z_{02} (Volume 2) at atmospheric pressure to introduce more air mass into the system through the opened valve for the next compression-expansion cycle.

Similarly, the stiffness of the air spring is given by:

$$k(z,z_0) = \frac{\partial^2 V}{\partial z^2}(z,z_0) = \frac{F_{\text{atm}} z_0}{z^2}, \quad \forall z \in [z_{\text{min}}, z_0]. \tag{17}$$

The stiffness of the spring depends on the length of the air cylinder z_0, and therefore, the air mass inside the cylinder. Increasing z_0 leads to smaller minimum stiffness and larger maximum stiffness:

$$k_{\text{min}} = k(z_0,z_0) = \frac{F_{\text{atm}}}{z_0}, \quad k_{\text{max}} = k(z_{\text{min}},z_0) = \frac{F_{\text{atm}} z_0}{z_{\text{min}}^2}. \tag{18}$$

Also, the range of achievable stiffness values increase as the active length of the air cylinder increases:

$$\frac{\partial(k_{\text{max}} - k_{\text{min}})}{\partial z_0} = F_{\text{atm}} \left(\frac{1}{z_{\text{min}}^2} - \frac{1}{z_0^2} \right) \geq 0, \quad \forall z_0 \in [z_{\text{min}}, z_{\text{max}}]. \tag{19}$$

Finally, we define the maximum elastic potential energy that can be stored by the air spring:

$$V_{\text{max}}(z_0) = F_{\text{atm}} \left(z_{\text{min}} - z_0 - \ln\left(\frac{z_{\text{min}}}{z_0} \right) \right). \tag{20}$$

In the next section, we will experimentally confirm the behavior of the air spring as predicted by (16), (17) and (20).

B. Experimental Setup and Procedure

Figure 2 shows the air spring used in our experiment. The device is composed of a double-acting air piston, where air can enter from both sides of the piston, with a bore diameter of 3 inches and a total stroke length of 12 inches. The air inlet on the compressed side is actuated by a valve, which lets in air at atmospheric pressure when opened. The change in air mass in the spring is facilitated by opening the valve when the air spring is at equilibrium, in order to minimize the work needed to move the piston. The minimum height of the compressed air chamber is 4.31 inches.

The spring was placed in an Instron 5944 testing machine to ensure consistent and precise force and displacement measurements throughout the course of the experiment. The testing instrument was programmed to ensure that the minimum value, i.e. $z = z_{\min}$, remained constant throughout each experimental trial. After each trial, the air flow control valve of the chamber was opened, and the shaft of the air spring was extended at atmospheric pressure. This process does not require any additional work in theory, but in practice, work needs to be done to compensate for the gravitational force acting on the piston, energy dissipation due to friction, and transient thermodynamic effects. Once the piston was raised up to the next testing height, the valve was closed, and the next trial of compression was performed. This process, shown in Fig. 2, was repeated at constant deformation rate until the compression length reached its maximum. The stroke lengths were between 1.5, 3.5, 5.5, and 7.3 inches, and the frequency of the deformation was consistent across trials to mitigate unaccounted thermodynamic effects.

C. Results

The experimental force-displacement curves starting from the equilibrium position z_0, and ending at the minimal height z_{min}, are shown in Fig. 3. These experimental force-deformation curves were fit with the normalized force function, found in (16). We obtained $R^2 > 99\%$ for all fits. The results confirm the previously discussed theoretical air spring model. Please see the correspondence between the measured (solid) lines and the fitted (dashed) lines in Fig. 3.

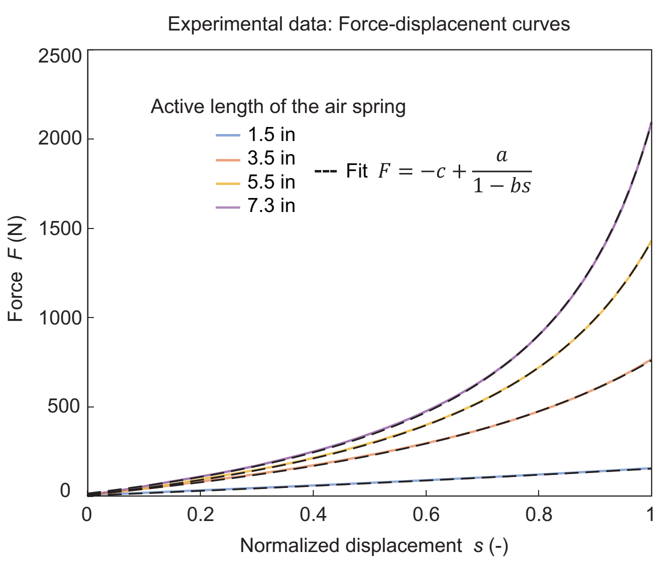

Fig. 3. Experimental result of the spring force (N) versus the normalized deformation of the spring $s = (z - z_{min})/(z_0 - z_{min})$. Four compression trials were performed with cylinder stroke lengths of 1.5, 3.5 5.5, and 7.3 inches measured from the same lowest compression height z_{min}. The deformation was normalized with the corresponding $z_0 - z_{min}$ value for each trial.

The data illustrates that as the active length of the cylinder was increased to include more air mass into the cylinder, larger force was needed to deform the piston to the same minimum height. Additionally, as the cylinder includes more air for each trial, the resultant force-deformation curve shows increased *stiffness* (see the slope of each curve) and increased *potential energy* (see the area underneath each curve). Minimal effort was needed to raise the piston to each new height since the pressure inside the cylinder was the atmospheric pressure while the valve was open. This way of changing the stiffness is consistent with our theory of requiring negligible amount of work to change the stiffness of the spring. These observations are consistent with the theoretical predictions, (16) and (17).

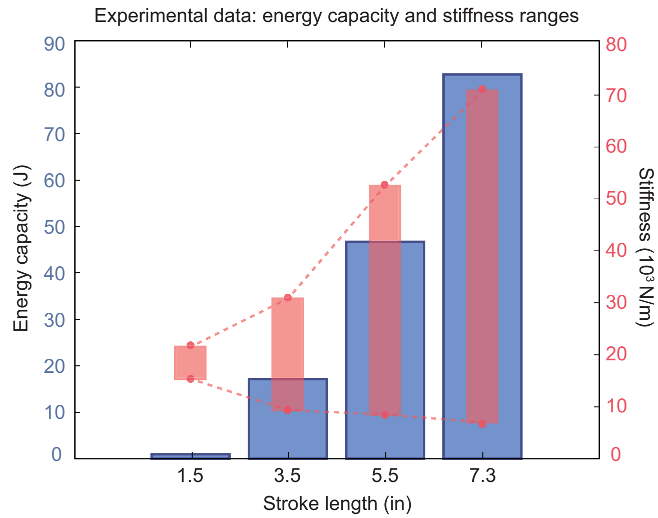

Fig. 4. Experimental results of the range of stiffness and the energy storage capacity of the air cylinder, for different equilibrium height, z_0, and stroke length. The maximum energy storage capacity and the stiffness modulation range both increased as more air was added into the cylinder.

Figure 4 more directly shows that the stiffness range and the energy storage capacity of the spring simultaneously increase, as the active length of the cylinder increases. This experimental result validates our theoretical conditions, predictions (18), (19), (20), and shows the adequacy of the simple air-spring model for the experimental investigation conducted in this paper.

VI. Conclusion

This paper defines analytical conditions to characterize variable stiffness springs that can simultaneously increase stiffness and energy storage capacity. These conditions led to the design of a controllable-volume air spring that experimentally validated our theoretical findings.

In some applications, air springs may be limited by the weight of the air cylinder, transient effects, and air leakage due to inadequate sealing and pressurization issues. Maintaining atmospheric pressure when changing stiffness requires the size of the cylinder to be scaled by the load. Pressurizing the cylinder to downsize for higher power output applications requires work to be done to change stiffness and increases start-up force. Finally, transient thermodynamic effects may add a scaling factor on the force function that is not captured by our isothermal assumption and a time constant for achieving ambient pressure equilibrium. Additionally, in this paper, a conceptual process was tested in a controlled laboratory setting, but a customized mechanical design may be required for successful practical implementation.

We note that despite the aforementioned limitations, variable volume air springs could be used to minimize the trade-off between increased stiffness and reduced deformation limit, which is the pitfall of low energy capacity. For example, in passive prosthetic devices, air springs may provide a larger range of joint stiffness without compromising the amount of energy that can be stored by the spring. In human jumping, weight-bearing and running augmentation, air springs may serve as a quasi-passive energy reservoir that could allow a large amount of energy to be first stored and then used to bypass the natural limitation of the biological limb [9], [18]. Furthermore, when used as part of an actuator, the variable stiffness spring may be able to recycle energy and reduce motor power. In this way, variable stiffness springs could reduce the size and weight of conventional high-output power actuators [21].

VII. Acknowledgment

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