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Multi-agent Formation Control based on Distributed Estimation with Prescribed Performance

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***Abstract*— We consider the distributed simultaneous estimation and formation control problem for swarms of identical mobile agents with limited communication, sensing and computation capabilities. In particular, we develop a novel scalable algorithm that encodes the formation specifications of the swarm via geometric moment statistics, which are estimated by a distributed scheme with prescribed performance guarantees. Based on the locally available information, each agent calculates an estimate of the global formation statistics, which is then employed by its local motion controller, thus creating a feedback interconnection between the estimator and the controller. The proposed scheme guarantees convergence of the global formation statistics to the desired values, while decoupling the estimation performance from the control performance. Moreover, a minimum allowable inter-agent distance can be predetermined so that inter-agent collision avoidance is achieved. Finally, simulation paradigms are provided to validate the approach.**

I. Introduction

During the last decades, a considerable amount of research has been focused on the control of networked multi-agent systems, which have induced a positive effect on our everyday lives via their adaptability and multi-functionality. Unfortunately though, the centralized approach to multi-agent control, in which a single agent has access to all agents' local information and issues control commands to them, is not robust to faults and does not scale well as the number of agents increases. Hence, distributed multi-agent control, which allows a group of agents, subject to potentially limited sensing and communication capabilities, to achieve global tasks without the intervention of a central controller or access to global information, has been rapidly advancing, proving its true potential through various applications. Examples of such applications can be found in formation control [1]–[6], sensor fusion [7]–[9], network connectivity maintenance [10], convex optimization [11]–[13], etc. In particular, within the realm of distributed multi-agent control, the distributed estimation-based formation control problem, where local controllers employ local estimates of global characteristics of a swarm in order to drive it to a desired configuration, plays an important role, e.g., the distributed control of a swarm to move along a specified path [14].

The formation control problem has been extensively studied over the past several years and many ways of describing a formation of a multi-agent system have been proposed. The majority of the existing results have concentrated on control with precise formation objectives, such as position-based formation control where a swarm formation is represented by the agents' relative positions [2]–[5]. Recent work in [6] introduced a position- and orientation-based formation control scheme for multiple rigid bodies, which achieves collision avoidance and connectivity maintenance. Nevertheless, the aforementioned approaches [2]–[6] do not scale well, since for a large number of agents the communication overhead becomes quite high and the extraction of the formation specifications is almost impossible in practical applications. Alternatively, the authors in [1] encode a swarm formation via a finite number of geometric moments, thus ensuring the scalability of the simultaneous estimation and formation control protocol they proposed. The PI estimator presented in [1] guarantees average tracking of its inputs with zero steady state error only under constant inputs; thus its tracking performance deteriorates for time-varying inputs. However, in formation control the inputs, namely the actual local moments in this case, change in time since the agents need to move to achieve a desired formation. This disadvantage of the PI estimator combined with the interconnection of the local estimators and controllers in [1] implies that even small estimation errors are likely to cause large estimation and formation errors in time. Additionally, due to the interdependence of the estimation and control performance in [1], specific small-gain conditions are employed to guarantee convergence to the desired global moment statistics as a function of the aggressiveness of the motion control. However, selecting appropriate gains to meet specific small-gain conditions is a rather complex task, thus reducing inevitably the algorithm's applicability.

In this paper, we introduce a distributed simultaneous estimation and formation control algorithm for multi-agent systems under undirected communication topologies, that guarantees convergence of the swarm formation statistics to the desired values. Moreover, the proposed scheme achieves inter-agent collision avoidance, by imposing a predefined lower bound on each inter-agent distance. The main contributions of this work include: i) avoidance of large estimation errors that would risk accurate convergence to the desired swarm formation statistics owing to the estimator's capability of achieving practical zero steady state error even for time-varying inputs, and ii) straightforward control gains selection, by decoupling the estimation performance from the control owing to robust estimation guarantees.

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II. Problem Formulation

Suppose that a swarm consists of a group of n mobile agents that have only local access to information. Specifically, the agents can access solely information of their own and their neighboring agents' state, and not of the whole swarm in general. We consider that the agents have limited communication capabilities in the sense that they can wirelessly communicate with only a few of the rest of the agents, which are their neighbors. Particularly, we assume that whether two agents can wirelessly communicate with each other is described by a fixed, undirected and connected graph $\mathcal{G}$ (communication graph), with the neighborhood set of the i-th agent denoted by N_i, $i \in \{1, ..., n\}$. We also assume that the communication radius is infinite, namely that wireless communication between two neighboring agents is possible no matter how large their distance is. Thereby, we ensure that the communication graph remains unchanged in time and thus connected as well. Finally, the agents' computation capabilities (i.e., the speed with which the agents can perform computations) are assumed to be the minimal required for the proper function of the algorithm.

In order to describe the swarm formation, we use first- and second-order moments. In particular, the first-order moments determine the centroid position of the swarm, whereas the second-order moments encode the orientation as well as the distribution of the swarm along its principal axes, which corresponds to its shape.

Let $p_i \in \mathbb{R}^r, \forall i \in \{1, ..., n\}$ denote the position of the i-th agent and $p = [p_1^T \; ... \; p_n^T]^T \in \mathbb{R}^{rn}$ the overall position vector of the swarm. To define the moment statistics mathematically, suppose that $\mathbf{triu} : \mathbb{R}^{r\times r} \to \mathbb{R}^{r(r+1)/2}$ denotes the mapping which starting from the main diagonal of a matrix, introduces its main and upper diagonals into a vector. For example, for $r = 3$ we get:

$$\mathbf{triu}\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right) = \begin{bmatrix} a_{11} \; a_{22} \; a_{33} \; a_{12} \; a_{23} \; a_{13} \end{bmatrix}^T . \tag{1}$$

As in [1], the moment-generating function $\phi : \mathbb{R}^r \to \mathbb{R}^L$, where $L = r(r+3)/2$, is defined as:

$$\phi(p_i) = \begin{bmatrix} p_i \\ \mathbf{triu}(p_i p_i^T) \end{bmatrix} . \tag{2}$$

Hence, the swarm configuration can be described using a vector function $f : \mathbb{R}^{rn} \to \mathbb{R}^L$, given by:

$$f(p) = \frac{1}{n}\sum_{i=1}^{n} \phi(p_i). \tag{3}$$

Suppose that $f^* \in Im(f)$ denotes the vector of the desired swarm formation statistics. Furthermore, consider that each agent has a point kinematic model defined by the single integrator:

$$\dot{p}_i = u_i \tag{4}$$

where u_i is the velocity control input. Each agent needs to know its position p_i to calculate its actual local moments $\phi(p_i)$, and the desired formation vector f^* to contribute to the convergence of the actual swarm formation statistics $f(p)$ to the desired values. Therefore, we consider that each agent has access to information of the form:

$$z_i = M(p_i, f^*). \tag{5}$$

Let us also define the potential function $J : \mathbb{R}^{rn} \to \mathbb{R}_+$ that encapsulates the desired swarm formation specifications and is given by:

$$J(p) = [f(p) - f^*]^T \, \Gamma \, [f(p) - f^*] , \tag{6}$$

where $\Gamma \in \mathbb{R}^{L\times L}$ is an appropriately selected symmetric positive definite gain matrix.

The goal of this work is to design, under the aforementioned network model, a collection of local controllers that allow the agents to move so that $f \to f^*$. In particular, we seek to determine a local controller K, a signal generator G and a distributed estimator Q-R of the form:

$$u_i = K(p_i, z_i, x_i, y_i, S_i) \tag{7}$$

$$s_i = G(p_i, z_i, x_i, y_i, S_i) \tag{8}$$

$$\dot{x}_i = Q(p_i, z_i, x_i, y_i, S_i) \tag{9}$$

$$y_i = R(p_i, z_i, x_i, S_i), \tag{10}$$

where s_i denotes the signal vector sent by agent i to its neighbors, S_i the collection of signals agent i receives from its neighbors, y_i the local estimate of the swarm's formation moments and x_i the internal state of the local estimator [1], which combined together achieve the minimization of J.

III. Main Results

A. Distributed Estimation with Prescribed Performance

In this subsection, we present a distributed estimation scheme that allows a group of agents to track the average of individual time-varying reference signals. Following [15], suppose that each agent implements the nonlinear dynamic average consensus estimator:

$$\begin{aligned} \dot{w}_i = & -k_r(w_i - v_i(t)) + \dot{v}_i(t) \\ & - k \sum_{j\in N_i} \rho_{ij}^{-1}(t) J_T\left(\tfrac{w_i - w_j}{\rho_{ij}(t)}\right) T\left(\tfrac{w_i - w_j}{\rho_{ij}(t)}\right) \end{aligned} \tag{11}$$

for all $i \in \{1, ..., n\}$ with k_r, $k > 0$, where $v_i : \mathbb{R}_+ \to \mathbb{R}$ is the local reference signal (input to the estimator), $w_i(t) \in \mathbb{R}$ is the local estimate of the average reference signal $v_{avg}(t) = \frac{1}{n}\sum_{i=1}^{n} v_i(t)$, $T : (-1, 1) \to (-\infty, \infty)$ is a strictly increasing, symmetric and bijective mapping, $J_T(\cdot)$ is the Jacobian (derivative) of the mapping $T(\cdot)$ that is strictly positive by construction, and $\rho_{ij}(t)$, $i \in \{1, ..., n\}$ and $j \in N_i$, denote certain decreasing functions of time that incorporate the desired transient and steady state consensus specifications for the consensus errors $w_i - w_j$, called performance functions [16]. Notice that the estimator in (11) is different from the one proposed in [15] in the sense that it involves the extra term $\dot{v}_i(t)$ on the right-hand side that allows practically convergence of the estimation error to zero. We adopt the mapping $T(\xi) = \frac{1}{2}\ln\left(\frac{1+\xi}{1-\xi}\right)$

as well as exponential performance functions of the form $\rho_{ij}(t) = (\rho_{ij}^0 - \rho_f)e^{-lt} + \rho_f$, where ρ_{ij}^0, l, ρ_f denote positive parameters selected appropriately such that $\rho_{ij}^0 = \rho_{ji}^0 > |w_i(0) - w_j(0)|$ and l, ρ_f encapsulate the desired transient and steady state specifications, respectively. Invoking the properties of the mapping $T(\cdot)$, it can be easily verified that consensus with predefined transient and steady state performance may be simply achieved by securing the boundedness of the transformed consensus error $T\left(\frac{w_i - w_j}{\rho_{ij}(t)}\right)$, since it directly leads to $|w_i(t) - w_j(t)| < \rho_{ij}(t)$, $\forall t \geq 0$. Consequently, the constant $\rho_f := \lim_{t\to\infty} \rho_{ij}(t)$ represents the maximum allowable size of the consensus error $w_i(t) - w_j(t)$ at the steady state, which may be set arbitrarily small thus achieving practical convergence to zero, whereas the decreasing rate of $\rho_{ij}(t)$, which is affected by the constant l in our case, introduces a lower bound on the required speed of convergence. Therefore, the appropriate selection of the performance functions imposes performance characteristics on the response of the consensus errors $w_i(t) - w_j(t)$. Finally, notice that contrary to [15], the addition of $\dot{v}_i(t)$ in (11) eliminates the estimation error of the input's derivative. Thus, Theorem 1 in [15] easily leads to the following proposition.

Proposition 1. *Consider bounded and continuous reference signals $v_i(t)$, $i \in \{1, ..., n\}$, with bounded first derivatives. Under a connected and undirected communication graph $\mathcal{G}$, the distributed estimator (11) achieves average consensus, with exponential rate equal to $\min\{l, k_r\}$, that satisfies:*

$$|w_i(t) - v_{avg}(t)| \leq \frac{Diam(\mathcal{G})}{2}\rho_f$$

for all $i \in \{1, ..., n\}$ at the steady state.

B. Formation Control

As indicated by (3), the swarm formation statistics are given by the average of the local moments. Hence, the agents can use the aforementioned dynamic average consensus scheme to estimate the actual swarm formation statistics $f(p) \in \mathbb{R}^L$, with their actual local moments $\phi(p_i) \in \mathbb{R}^L$ as inputs. Specifically, each agent employs L estimators as follows:

$$\begin{aligned} \dot{x}_{il} =& -k_r(x_{il} - \phi_l(p_i)) + [\nabla\phi_l(p_i)]^T \dot{p}_i \\ & - k \sum_{j\in N_i} \rho_{ij}^{-1}(t) J_T\left(\tfrac{x_{il} - x_{jl}}{\rho_{ij}(t)}\right) T\left(\tfrac{x_{il} - x_{jl}}{\rho_{ij}(t)}\right) \end{aligned} \tag{12}$$

$$y_{il} = x_{il} \tag{13}$$

for all $l \in \{1, ..., L\}$ and $i \in \{1, ..., n\}$. From (12) it can be deduced that each agent has to send its own estimate of the actual swarm formation statistics to its neighbors. Therefore, each agent employs noise-free measurements and reference information of the form:

$$z_i = \left[p_i^T \;\; f^{*T}\right]^T \tag{14}$$

and its signal generator is given by:

$$s_i = y_i, \tag{15}$$

for all $i \in \{1, ..., n\}$. We want to design a control law following the gradient of the potential function J defined in (6). Such gradient law guarantees convergence of the actual swarm position vector to the set $J_{cr} = \{p \in \mathbb{R}^{rn} : \nabla J(p) = 0\}$, which contains the critical points of J. However, an unwanted critical point p of J (i.e., a point p where $\nabla J(p) = 0$ and $f(p) \neq f^*$) may raise by the gradient flow $-\nabla J(p)$. To ensure convergence of $f(p)$ to f^*, we need to ascertain that the swarm will not get stuck at such points. Hence, given a compact set of swarm configurations $\mathcal{D} \subset \mathbb{R}^{rn}$ and a goal vector $f^* \in f(\mathcal{D})$, we define the cone of all symmetric positive definite matrices Γ such that no unwanted local minima of $\Gamma\nabla J$ lie in $\mathcal{D}$ as in [1]. Then, we construct the following local gradient controller:

$$\begin{aligned} \dot{p}_i = u_i &= -[\mathbb{I}_{r\times r} + D_i^T D_i]^{-1} \left.\frac{\partial J}{\partial p_i}\right|_{f=x_i} \\ &= -[\mathbb{I}_{r\times r} + D_i^T D_i]^{-1} D_i^T \Gamma [x_i - f^*], \end{aligned} \tag{16}$$

where $D_i = \mathcal{J}\phi(p_i)$, with $\mathcal{J}\phi(\cdot)$ denoting the $L \times r$ Jacobian matrix of ϕ as defined in (2) and $[\mathbb{I}_{r\times r} + D_i^T D_i]^{-1}$ scaling the control signal. Thus, the idea is to ensure that all unwanted critical points are saddles.

Theorem 1. *Every trajectory of the swarm dynamics (12) and (16) is bounded in forward time and its positive limit set consists of equilibria, as long as we select sufficiently small ρ_f. Moreover, there exists a matrix Γ such that all unwanted critical points become saddles.*

Proof. Let us first define the vectors $d_\phi = [d_{\phi_1}^T \; ... \; d_{\phi_n}^T]^T$ and $e_f = [e_{f_1}^T \; ... \; e_{f_n}^T]^T$, where:

$$d_{\phi_i} = \frac{d}{dt}\phi(p_i) \tag{17}$$

$$e_{f_i} = f(p) - x_i, \tag{18}$$

with e_{f_i} denoting the i-th agent's estimation error vector of the actual swarm formation statistics, for all $i \in \{1, ..., n\}$. The proof proceeds in two phases. Firstly, we prove that for sufficiently small ρ_f, every swarm trajectory is bounded over time and its positive limit set contains only equilibria. Then, we ensure that the actual swarm formation statistics converge to the desired values, by showing that the swarm leaves configurations that correspond to unwanted critical points after a small perturbation.

Phase A. Employing Proposition 1 as well as the Converse Lyapunov Theorem [17], it is deduced that there exists a proper function $U(e_f)$ that satisfies the inequality:

$$\dot{U} \leq -\min\{l, k_r\}U + a\left(d_\phi, \tfrac{1}{\rho_f}\right) \tag{19}$$

where a is a $\mathcal{KL}$-function. Invoking the Mean Value Theorem, we obtain:

$$\left.\tfrac{\partial J}{\partial f^j}\right|_{f=f(p)} - \left.\tfrac{\partial J}{\partial f^j}\right|_{f=x_i} = H_{ij} e_{f_i} \tag{20}$$

where $H_{ij} = \left.\frac{\partial^2 J}{\partial f^j \partial f^i}\right|_{f=\tilde{f}_{ij}(p)}$ with $\tilde{f}_{ij} = f - \alpha_j(f - x_i)$ for some $0 \leq \alpha_j \leq 1$. Thus, the effect of the estimation error in the control effort is given by:

$$\left.\frac{\partial J}{\partial p_i}\right|_{f=f(p)} - \left.\frac{\partial J}{\partial p_i}\right|_{f=x_i} = \left(\frac{\partial f}{\partial p_i}\right)^T \left(\left.\frac{\partial J}{\partial f}\right|_{f=f(p)} - \left.\frac{\partial J}{\partial f}\right|_{f=x_i}\right) = \frac{1}{n} D_i^T H_i e_{f_i}, \tag{21}$$

where $H_i = [H_{i1}^T \; ... \; H_{iL}^T]^T$ denotes the Hessian matrix of the system. Differentiating (6) with respect to time and substituting (16)-(18) and (21), we get:

$$\begin{aligned} \dot{J} &= \sum_{i=1}^n \dot{p}_i^T \left.\frac{\partial J}{\partial p_i}\right|_{f=f(p)} \\ &= \sum_{i=1}^n \dot{p}_i^T [\mathbb{I}_{r\times r} + D_i^T D_i][\mathbb{I}_{r\times r} + D_i^T D_i]^{-1} \left(\left.\frac{\partial J}{\partial p_i}\right|_{f=x_i} + \frac{1}{n} D_i^T H_i e_{f_i}\right) \\ &= -\sum_{i=1}^n \dot{p}_i^T [\mathbb{I}_{r\times r} + D_i^T D_i]\dot{p}_i + \frac{1}{n}\sum_{i=1}^n d_{\phi_i}^T H_i e_{f_i} \\ &= -\sum_{i=1}^n \dot{p}_i^T \dot{p}_i - \sum_{i=1}^n d_{\phi_i}^T d_{\phi_i} + \frac{1}{n}\sum_{i=1}^n d_{\phi_i}^T H_i e_{f_i} \\ &= -\dot{p}^T \dot{p} - d_\phi^T d_\phi + \frac{1}{n} d_\phi^T H e_f \end{aligned} \tag{22}$$

where H is a block diagonal matrix with elements H_i, $i \in \{1, ..., n\}$.

To proceed, consider the storage function:

$$V(p, e_f) = J(p) + \gamma U(e_f) \tag{23}$$

where γ is a positive constant. Differentiating V with respect to time and invoking (19) and (22), we get:

$$\begin{aligned} \dot{V} \leq & -\dot{p}^T \dot{p} - d_\phi^T d_\phi + \frac{1}{n} d_\phi^T H e_f \\ & - \gamma \min\{l, k_r\} U(e_f) + \gamma a\left(d_\phi, \tfrac{1}{\rho_f}\right). \end{aligned} \tag{24}$$

Finally, completing the squares in (24), we arrive at:

$$\begin{aligned} \dot{V} \leq & -\dot{p}^T \dot{p} + \tfrac{1}{4n^2}||He_f||^2 - \gamma \min\{l, k_r\} U(e_f) + \\ & \gamma a\left(d_\phi, \tfrac{1}{\rho_f}\right) \end{aligned} \tag{25}$$

which by selecting sufficiently large γ and very small ρ_f yields $\dot{V} \leq 0, \forall t \geq 0$, since a is decreasing with respect to $\frac{1}{\rho_f}$. Hence, $V(t)$ is nonincreasing along the trajectories of (12) and (16) over time. Notice that no small-gain conditions or other assumptions were required for $V(t)$ to be nonincreasing, except for choosing a sufficiently small value for ρ_f. This is achieved thanks to the increased robustness of the proposed estimator that allows decoupling the estimation performance from the control. Specifically, the estimator guarantees estimate consensus and average tracking with exponential convergence rate and practical zero steady state error regardless of the bound of the control input. Therefore, although the local estimators and controllers form interconnected systems, the estimation and control gains can be regulated independently.

Since $J(p)$ and $U(e_f)$ are proper, then $V(p, e_f)$ is also proper and all signals are bounded. Furthermore, invoking LaSalle's theorem, it is deduced that as $t \to \infty$ every trajectory approaches its positive limit set L^+, which is nonempty, compact, connected and contains only equilibria of the form $(p, e_f) = (p^*, 0)$ for some $p^* \in J_{cr}$. Consequently, V and J are constant on every positive limit set, which implies that all equilibria contained in positive limit sets with at least one bad equilibrium should be bad.

Phase B. Up to this point, what remains to be shown is that any positive limit set containing bad equilibria is strongly unsteady and thus unstable (in the sense of Lyapunov) and unattractive [1]. Particularly, we need to ensure that any trajectory that approaches such a limit set will leave a neighborhood of it, after a slight perturbation. Towards this direction, let $L_b^+ = L_0^+ \times \{0\} \times \{0\}$, where $L_0^+ \subset \mathcal{D} \cap J_{cr}$, denote a positive limit set consisting of bad equilibria. Notice that ϕ is polynomial and thus subanalytic, from which it is deduced that J is also subanalytic. Employing Theorem 10, stated in [1], we conclude that J is locally constant on J_{cr}, which implies that there exists an open neighborhood N_p such that J is constant on $cl(N_p) \cap J_{cr}$, for all $p \in L_0^+$. Let us define the open set $N = \bigcup\{N_p : p \in L_0^+\}$ as well as an open neighborhood of L_b^+, given by $\mathcal{O} = N \times \mathbb{R}^{rn} \times \mathbb{R}^{L\times n}$. By Lemma 11, stated in [1], it is obtained that J is constant on $cl(N) \cap J_{cr}$. Following [1], suppose that $\mathcal{U}$ denotes any open set that satisfies $L_b^+ \cap \mathcal{U} \neq \varnothing$, and let $(p^*, 0) \in L_b^+ \cap \mathcal{U}$. Since p^* is not a local minimum of J by assumption, there exists a point $(\bar{p}, 0) \in \mathcal{U}$ such that $J(\bar{p}) < J(p^*)$ and therefore also $V(\bar{p}, 0) < V(p^*, 0)$. If $L_1^+ \times \{0\} \times \{0\}$ is the positive limit set of the trajectory that starts from $(\bar{p}, 0)$, we can deduce that J is constant on L_1^+. Taking into consideration that V is nonincreasing, we may conclude that J has a smaller value on L_1^+ than on $cl(N) \cap J_{cr}$. Consequently, as in [1], since $L_1^+ \subset J_{cr}$, it is deduced that $L_1^+ \cap cl(N) = \varnothing$, which implies that the trajectory that starts from the point $(\bar{p}, 0)$ finally escapes $\mathcal{O}$. Hence, L_b^+ is strongly unsteady and thus all unwanted equilibria are saddle points. ■

Remark 1. *Notice that the estimator in (12) and the controller in (16) form a feedback interconnected system. In particular, the estimator receives u_i as input from the controller, while the controller receives x_i as input from the estimator. Nevertheless, and in contrast to [1], the performance of the estimator is decoupled from the performance of the controller, since the estimate consensus is robustly imposed by the performance functions and the algorithm guarantees both estimate consensus and average tracking with exponential convergence rate and practical zero steady-state error, regardless of the bound of the control input u_i. More specifically, increasing l and decreasing ρ_f will result in faster convergence and more accurate consensus, whereas increasing k_r and decreasing ρ_f will result in faster convergence and more accurate average tracking. Moreover, owing to the estimator's tracking performance,* *large estimation errors, that would risk accurate convergence to the desired swarm formation statistics, are guaranteed to be avoided, unlike in the case of [1]. Finally, regarding the controller gain matrix* Γ*, it can be easily computed following [1].*

Remark 2. *The proposed algorithm can be also extended to achieve formation control with inter-agent collision avoidance following [18], where a region-based shape controller for a swarm of robotic agents was proposed that guarantees the motion of the agents inside a desired region while maintaining a minimum inter-agent distance. In this case, the measurements' vector defined in (5) takes the form* $z_i = [p_i^T \; P_i^T \; f^{*T}]^T$*, where* P_i *denotes the positions of the agents whose distance from the* i*-th agent does not exceed the sensing radius* r_s*, or equivalently the agents that belong to agent* i*'s sensing neighborhood set, denoted by* N_{s_i}*. Thus, invoking (16), we modify the controller as:*

$$u_i = -[\mathbb{I}_{r\times r} + D_i^T D_i]^{-1} \frac{\partial J}{\partial p_i}\bigg|_{f=x_i} - k_c \frac{\partial Q_{Lij}(\Delta p_{ij})}{\partial \Delta p_{ij}} \quad (26)$$

where k_c *is a positive gain and* $Q_{Lij}(\Delta p_{ij})$ *is a potential function of the inter-agent distances defined as in [18] to secure a minimum allowable threshold, denoted by* r_c*. Finally, notice that it is required to choose a relatively large gain* k_c *in order to ensure that the specified* r_c *is achieved as long as it is compatible with the desired formation specifications.*

IV. Simulation Results

In this section, we present two simulation studies. The first one illustrates the effectiveness of the aforementioned theoretical findings. The second one demonstrates how the proposed algorithm can be applied to the motion control problem of a multi-agent system along a specified trajectory within an obstacle-cluttered workspace, even when considering position measurement noise and communication delays. In particular, consider a swarm of $n = 6$ agents moving on a plane and suppose the agents' communication network is represented by a fixed connected graph. Additionally, if the position of the i-th agent is denoted by $p_i = [p_{ix} \; p_{iy}]^T$, then the formation statistics of the swarm are given by the vector:

$$f(p) = \frac{1}{n}\sum_{i=1}^{n}[p_{ix} \; p_{iy} \; p_{ix}^2 \; p_{iy}^2 \; p_{ix}p_{iy}]^T.$$

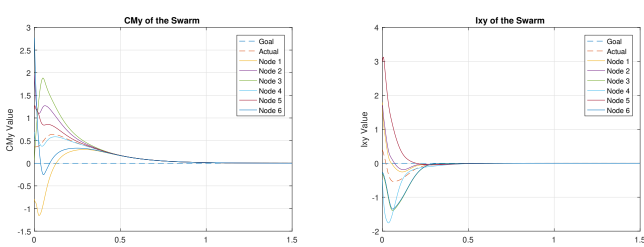

Fig. 1: The evolution of CM_y, I_{xy}.

Example A. We simulated the proposed algorithm in order to achieve the following swarm formation statistics $f^* = [0 \; 0 \; 5 \; 5 \; 0]^T$. The estimator parameters were set as $k = 4$, $k_r = 5$, $l = 10$, $\rho_f = 0.001$ and the controller gain matrix as $\Gamma = diag(5, 5, 5, 5, 5)$. We selected minimum allowable inter-agent distance $r_c = 1$ and gain $k_c = 150$.

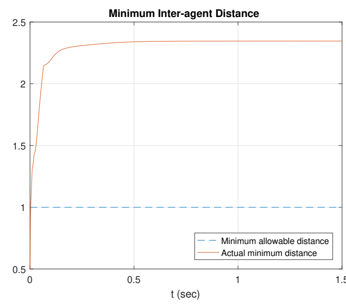

Fig. 2: The minimum inter-agent distance.

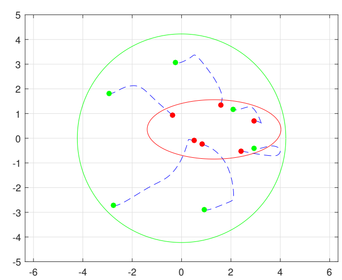

Fig. 3: The trajectories of the agents (the red and green points denote the initial and final positions of the agents respectively) and the uniform-density ellipses corresponding to the initial (red ellipse) and desired (green ellipse) swarm configurations.

Figures 1 and 3 verify the convergence guarantees of the proposed algorithm, whereas Figure 2 validates that inter-agent collision avoidance is accomplished. More specifically, in Figure 1 we present each agent's estimates of the moments CM_y (second component of f) and I_{xy} (fifth component of f), along with their actual and desired values. It can be easily observed that the proposed distributed scheme achieves convergence of the formation statistics to their desired values with practical zero steady state error, thus confirming the theoretical findings. As shown in Figure 2, the agents' motion respects the minimum allowable inter-agent distance over time; thus inter-agent collision avoidance is ensured. This is indicated by the fact that the minimum inter-agent distance remains greater than r_c over time, although this happens after a while given that the initial distances between some pairs of agents are smaller than r_c. Finally, Figure 3 illustrates the trajectories of the agents as they move from their initial positions to their final ones. It is verified that the desired formation vector is achieved, since the final positions of the agents (represented by green points) form a circle (as expected since $I_{xx}^* = I_{yy}^* = 5$ and $I_{xy}^* = 0$) with center (0,0) (as expected since $CM_x^* = CM_y^* = 0$) and radius calculated from $I_{xx}^* = I_{yy}^* = 5$ through specific transformations. To demonstrate this clearer, in Figure 3 we present the initial and desired swarm formations using the uniform-density ellipses with the respective first- and second-order moments.

Example B. The motion of a swarm along a certain path within an obstacle-cluttered environment may be described by successive configurations of the swarm along the path, using first- and second-order moments. In particular, the first-order moments, which determine the center of mass of the swarm, may be used to represent distinct points of the desired trajectory, whereas the second-order moments, which specify the shape and orientation of the swarm, may be selected so that obstacle avoidance is achieved. Thus, applying the proposed scheme to attain appropriately defined successive formation specifications guarantees safe motion of the swarm along the specified trajectory without any collisions.

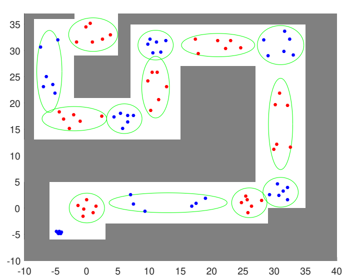

Fig. 4: Successive swarm configurations (with blue and red alternately) with the corresponding desired uniform-density ellipses during the motion of the swarm along a specified path.

Suppose that we want the center of mass of the aforementioned swarm, which initially coincides with $(-4.4, -4.7)$, to arrive at $(1, 33)$, through the motion of the swarm inside the configuration space illustrated in Figure 4 (obstacles are depicted with gray and free space with white). First, we designed a feasible path and then specified the corresponding formation statistics. The parameters of the algorithm were chosen as in Example 1, except Γ, which was selected as explained in [1] for each desired formation, and $r_c = 0.7$. To make the simulation more realistic, we considered position measurement noise of normal distribution with zero mean value and standard deviation 2 cm and communication delay of 10 msec. Figure 4 shows the initial swarm configuration (initially the agents are very close to each other), as well as the configurations corresponding to successive desired formations. We have also provided the uniform-density ellipses that correspond to all desired formations. It can be concluded that the agents are capable of following the desired path without colliding with each other or the obstacles of the configuration space.

V. Conclusions

We proposed a scalable framework for multi-agent distributed simultaneous estimation and formation control with inter-agent collision avoidance. We proved that the algorithm guarantees convergence of the swarm formation statistics to the desired values and ensures avoidance of large estimation errors that would risk accurate convergence. Moreover, we were able to decouple the estimation from the control performance, thus simplifying the control gains selection. Simulation results were provided to verify the theoretical findings and demonstrate the applicability of the developed scheme. Future research efforts will be devoted towards studying the case of finite communication radius, which implies time-varying communication graphs and thus the need for connectivity maintenance specifications.

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